MATHEMATICS

Maximum Marks: 80
Time Allotted: Three Hours
Reading Time: Additional Fifteen Minutes

Instructions to Candidates

1. You are allowed an **additional fifteen minutes** for **only** reading the paper.
2. You must **NOT** start writing during reading time.
3. The question paper has **11 printed pages**.
4. It consists of **20 questions and four sections: A, B, C and D**. **All questions are compulsory.**
5. **Section A** comprises **very short answer questions** of **1 mark** each.
6. **Section B** consists of **short answer questions** of **2 marks** each.
7. **Section C** consists of **moderately long answer questions** of **3 marks** each.
8. **Section D** consists of **long answer questions** of **5 marks** each.
9. Internal choices have been provided in **three** questions, each in **Sections B, C and D.**
10. While attempting **Multiple Choice Questions in Section A**, you are required to **write only ONE option as the answer.**
11. The intended marks for questions or parts of questions are given in the brackets [].
12. All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
13. Mathematical tables and graph papers are provided.

Instruction to Supervising Examiner

1. Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Note: The Specimen Question Paper in the subject provides a realistic format of the Board Examination Question Paper and should be used as a practice tool. The questions for the Board Examination can be set from any part of the syllabus, though the format of the Board Examination Question Paper will remain the same as that of the Specimen Question Paper. The weightage allocated to various topics, as given in the syllabus, will be strictly adhered to.

SECTION A – 20 MARKS

Question 1

In subparts (i) to (xvii) choose the correct options and in subparts (xviii) to (xx), answer the questions as instructed.

(i) Additive inverse of i is expressed as $a + i\,b$, then $b =$ **(Recall)** **[1]**

(a) 0

(b) 1

(c) -1

(d) $-i$

(ii) The centre of the circle $2x^2 + 2y^2 - 4x - 8y - 45 = 0$ is: **(Recall)** **[1]**

(a) $(2, 4)$

(b) $(-2, -4)$

(c) $(-1, -2)$

(d) $(1,2)$

(iii) The number of terms in the expansion of $\left(\mathrm{a}^2 - 2\frac{b}{a^{-1}} + \mathrm{b}^2\right)^{10}$ is: **[1]**
(Understanding)

(a) 20

(b) 15

(c) 21

(d) 11

(iv) The equation of line intersecting the x-axis at a distance of 3 units to the left of origin with slope -2 is: **(Understanding)** **[1]**

(a) $2x + y + 6 = 0$

(b) $2x - y + 6 = 0$

(c) $-2x + y + 6 = 0$

(d) $2x - y - 6 = 0$

(v) The Cartesian product A × A has 16 elements among which are (0,2) and (1,3). Which of the following statements is correct? **(Understanding)** **[1]**

Statement I: It is possible to determine the set A.

Statement II: A × A contains the element (3,2).

(a) Both the statements are true.

(b) Both the statements are false.

(c) Statement I is true and Statement II is false.

(d) Statements I is false and Statement II is true.

(vi) Which one of the following is **NOT** true in the III$^{rd.}$ quadrant? **(Recall)** **[1]**

(a) $\tan\theta$ increases from 0 to ∞

(b) $\sin\theta$ increases from 0 to -1

(c) $\cot\theta$ decreases from ∞ to 0

(d) $\cos\theta$ decreases from -1 to 1

(vii) Variance of the data 2, 4, 5, 6, 8, 17 is 23·33. Then variance of 4, 8, 10, 12, 16, 34 will be: **(Understanding)** **[1]**

(a) 93·32

(b) 25·33

(c) 46·66

(d) 48·66

(viii) The eccentricity of the ellipse with foci at (± 5, 0) and $x = \frac{36}{5}$ as one of the directrices is: **(Recall)** **[1]**

(a) $\frac{5}{6}$

(b) $\frac{3}{5}$

(c) $\frac{6}{5}$

(d) $\frac{5}{9}$

(ix) Shown below is a solved differentiation problem: **[1]**

$y = sin^3 x . cos^3 x$

(Step 1) $= \frac{1}{8} sin^3 2x$

Differentiating w.r.t 'x' we get

(Step 2) $\Rightarrow \frac{dy}{dx} = \frac{3}{8} sin^2 2x \times \frac{d}{dx}(\sin 2x)$

(Step 3) $\Rightarrow \frac{dy}{dx} = \frac{3}{8} sin^2 2x . \cos 2x \times \frac{d}{dx}(x)$

(Step 4) $\Rightarrow \frac{dy}{dx} = \frac{3}{4} sin^2 2x . \cos 2x$

In which step there is an error in solving? **(Understanding)**

(a) Step 1

(b) Step 2

(c) Step 3

(d) Step 4

(x) For any two sets A and B, $(A - B) \cap$ B = **(Recall)** **[1]**

(a) B

(b) A

(c) Null set (φ)

(d) Universal set (ξ)

(xi) What is the equation of directrix of parabola $y^2 = 4px$, where $p < 0$ and $p^2 + p - 2 = 0$? **(Understanding)** **[1]**

(a) $x + 1 = 0$

(b) $x - 2 = 0$

(c) $x - 1 = 0$

(d) $x + 2 = 0$

(xii) The sum of the first n terms of a series S is $3n^2 + 5n$. Which one of the following statements is correct? **(Analysis)** **[1]**

(a) The terms of S form an arithmetic progression with common ratio 6.

(b) The terms of S form a geometric progression having first term 8.

(c) The terms of S form an arithmetic progression with common difference 14.

(d) The terms of S form an arithmetic progression with common difference 6.

(xiii) What is the number of selections of at most 3 things from 6 different things? **[1]**

(Application)

(a) 20

(b) 22

(c) 41

(d) 42

(xiv) If $\frac{-2}{x-3} > 0$, then x belongs to: **(Understanding)** **[1]**

(a) $(3, \infty)$

(b) $[3, -\infty]$

(c) $(-\infty, 3)$

(d) $(-\infty, 3]$

(xv) Consider the following in respect of the circle $4x^2 + 4y^2 - 4ax - 4ay + a^2 = 0$ **[1]**

(P) The circle touches both the axes.

(Q) The diameter of the circle is $2a$.

(R) The centre of the circle lies on the line $x + y = a$.

How many of the statements given above are correct? **(Understanding)**

(a) Only (P)

(b) Only (P) and (Q)

(c) Only (P) and (R)

(d) Only (Q) and (R)

(xvi) **Assertion (A)**: Cardinal number of the power set of {{0, {1}, 3}}= 8 **[1]**

Reason (R): If A is a set with n(A) = m, the n (P(A)) = 2^m

Which one of the following is correct? **(Recall)**

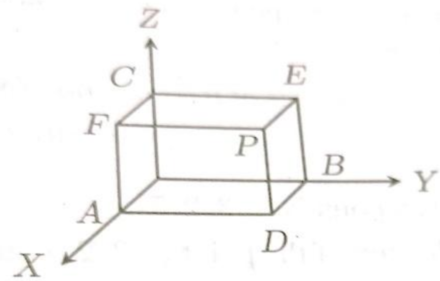
(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(xvii) Coordinate of the point P is (2, 4, 5). **[1]**



Assertion: The image of the point F in YZ plane is $(-2,4,5)$.

Reason: When reflecting a point (x, y, z) in the $\boldsymbol{XOY}$ **plane**, the Z-coordinate changes its sign. The reflected point becomes $(x, y, -z)$.

Which one of the following is correct? **(Understanding)**

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(xviii) Find the sum of the intercepts of a line whose normal from the origin has a length of 4 units and makes an angle of 15° with the positive X-axis. **(Application)** **[1]**

(xix) Find number of words that can be formed with the letters of the word '**LAHORE**', if $\boldsymbol{L}$ and $\boldsymbol{A}$ are always together. **(Understanding)** **[1]**

(xx) A function is defined by $f(x) = \pi + \sin^2 x$. Find the range of the function. **[1]**
(Recall)

SECTION B – 14 MARKS

Question 2 **[2]**

Imagine you are designing a straight ramp that needs to connect two points at different elevations. Point P is at a horizontal distance of 10 meters and a vertical height of 2 meters from the starting point Q.

Write the equation of the line representing the ramp, considering the starting point Q as the origin. What is the angle of inclination of the ramp? **(Application)**

Question 3 **[2]**

If a, b and c $(a > 0, c > 0)$ are in G.P, justify the following with respect to the equation $ax^2 + bx + c = 0$: **(Application)**

(i) The equation has imaginary roots.

(ii) The product of the roots of the equation $\frac{b^2}{a^2}$

Question 4 **[2]**

If $f(x) = \frac{xsinx}{1+cosx}$, then find $f'\left(\frac{\pi}{2}\right)$. **(Evaluate)**

Question 5 **[2]**

(i) If the sum of binomial coefficients in the expansion of $(x + y)^n$ is 256, then in which term of the expansion does the greatest binomial coefficient occurs? **(Understanding)**

OR

(ii) If the coefficient of $(r - 5)$th and $(2r - 1)$th terms in the expansion of $(1 + x)^{34}$ are equal, then find the value of r. **(Understanding)**

Question 6 **[2]**

(i) Does $\lim_{x \to 0} \frac{|sinx|}{x}$ exist? Give justification. **(Understanding)**

OR

(ii) Evaluate: $\underset{x \to \pi}{lt} \left(\frac{1 - Sin\frac{x}{2}}{Cos\frac{x}{2}\left[Cos\frac{x}{4} - Sin\frac{x}{4}\right]} \right)$ **(Understanding)**

Question 7 **[2]**

(i) Two finite sets have 'm' and 'n' elements. The number of subsets of the first set is 112 more than that of second set. Find the values of 'm' and 'n'. **(Recall)**

OR

(ii) If A and B are two sets and U is the universal set such that n(U) = 700, n(A) = 290, n(B) = 240 and n $(A \cap B)$= 110, then find n $(A' \cap B')$. **(Understanding)**

Question 8 **[2]**

Two cards are drawn at random without replacement from a deck of 52 cards. What is the probability that both the cards drawn are kings? **(Understanding)**

SECTION C – 21 MARKS

Question 9 **[3]**

$A\ (1,2,3), B\ (0,4,1), C(-1,-1,-3)$ are the vertices of a triangle ABC. Find the following: **(Understanding)**

(i) Length of AB and AC. **[1]**

(ii) Coordinates of the point in which the bisector of the ∠BAC meets BC. **[2]**

Question 10 **[3]**

Given that α and β are the roots of $qx^2 - px + r = 0$, form an equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$. **(Understanding)**

Question 11 **[3]**

Find the points of intersection of the line $y = x + 1$ and the circle $x^2 + y^2 = 5$. Hence or otherwise, find the length of the chord. **(Evaluate)**

Question 12 **[3]**

A plot of land is in the shape of a sector of a circle of radius 55m. The length of fencing that is erected along the edge of the plot to enclose the land is 176m. Calculate the area of the plot of land. **(Evaluate)**

Question 13 **[3]**

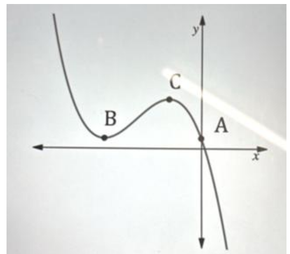
(i) If $2 - i\sqrt{3}$ where $i = \sqrt{-1}$ is a root of the equation $x^2 + ax + b = 0$, then find the value of $a\ + b$. **(Analysis)**

OR

(ii) If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then what is |z| equal to? **(Analysis)**

Question 14 **[3]**

(i) The curve G has equation $y = 1 - x^3 - 6x^2 - 9x$ and part of the graph of G is shown below:



(Analysis)

(a) Find the derivative of y with respect to x.

(b) If the tangents drawn to the curve at the points B and C are parallel to *x-axis*, then find the coordinates of B and C respectively.

OR

(ii) Find the derivative of $\sqrt{3x+4}$ by using first principle. **(Application)**

Question 15 **[3]**

(i) The alternative voltage V, in a domestic electrical circuit $'t'$ seconds after it is switched on is modelled by the function, $V = 115\,Sin\omega t + 115\sqrt{3}cos\omega t$.

Express $115\,Sin\omega t + 115\sqrt{3}cos\omega t$ in the form $RSin(\omega t + \alpha)$, where R and α are the constants to be found, R> 0 and α is acute. **(Understanding)**

OR

(ii) If A and B are two acute angles having its sum equal to $\frac{\pi}{4}$, then answer the following: **(Understanding)**

(a) Find the value of $(cotA - 1)(cotB - 1)$.

(b) Hence, find the value of $\cot\frac{\pi}{8}$.

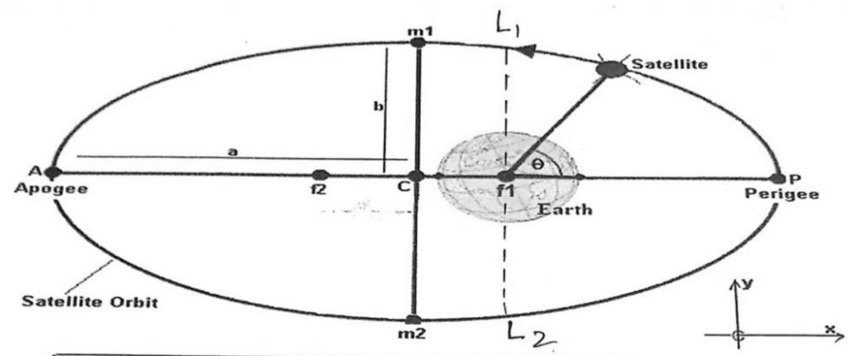
SECTION D – 25 MARKS

Question 16 **[5]**

(i) A focal chord of the parabola $y^2 = 8x$ makes an angle of 60° with the $x -$ axis. Find the length of this focal chord. **(Evaluate)**

OR

(ii) A satellite moves in an Elliptical orbit around the earth as shown in the figure below. It moves so that the sum of its distances from the points f2 (-2,0) and f1 (2,0) is 8 units. **(Evaluate)**



Satellite in an Elliptical Orbit Around the Earth

Based on the above information, find the following:

(a) The length of its semi-major axis of the elliptical orbit. **[1]**

(b) Eccentricity of the elliptical orbit. **[1]**

(c) Equation of the elliptical orbit. **[2]**

(d) Distance between f1 and P(Perigee). **[1]**

Question 17 **[5]**

(i) For the word '**DEVASTATION**', determine the number of 4-letter combinations and 4-letter permutations. **(Analysis)**

OR

(ii) The letters of the word '**CRICKET**' are written in all possible orders. **(Analysis)**

(a) Find the total number words with or without meaning. **[1]**

(b) If these words are written out as in a dictionary, find the rank of the word '**CRICKET**'. **[4]**

Question 18

(i) Evaluate: $\cos\alpha + \cos\left(\alpha + \frac{2\pi}{3}\right) + \cos\left(\alpha + \frac{4\pi}{3}\right)$. **(Application)** **[2]**

(a) Hence prove that: **[1]**

$cos^3\alpha + cos^3\left(\alpha + \frac{2\pi}{3}\right) + cos^3\left(\alpha + \frac{4\pi}{3}\right) = 3\cos\alpha\cos\left(\alpha + \frac{2\pi}{3}\right)\cos\left(\alpha + \frac{4\pi}{3}\right)$

(b) If $cos^3\alpha + cos^3\left(\alpha + \frac{2\pi}{3}\right) + cos^3\left(\alpha + \frac{4\pi}{3}\right) = \frac{k}{k+1}\cos k\alpha$, find k. **[2]**

OR

(ii) The function f is defined by $f(x) = x^2 - 6x + 17$. **[2]**

Sketch the graph of the function.

Hence or otherwise: **(Application)**

(a) Find the maximum / minimum value of the function. **[1]**

(b) Determine the sign of the function. **[1]**

(c) Find the range of $f(x)$. **[1]**

Question 19 **[5]**

Find the mean and standard deviation using the **deviation method** for the following:

Age	20-25	25-30	30-35	35-40	40-45	45-50	50-55
No of employees	170	110	80	45	40	30	25

Find the percentage of the employees in an organisation whose age lies between $\bar{x} - \sigma$ and $\bar{x} + \sigma$. **(Evaluate)**

Question 20 **[5]**

S_1,S_2,S_3,...,S_n are the sums of n infinite geometric progressions. The first terms of these progressions are 1, $2^2 - 1$, $2^3 - 1$, ..., $2^n - 1$ and the common ratios $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots, \frac{1}{2^n}$.
Find S_1+S_2+S_3+...+S_n. **(Evaluate)**

MATHEMATICS

ANSWER KEY

SECTION A – 20 MARKS

Question 1

In answering Multiple Choice Questions, candidates have to write either the correct option number or the explanation against it. Please note that only ONE correct answer should be written.

(i) (c) or -1 **[1]**

Additive inverse of $i = -i = 0 + (-1)i$.

$\therefore b = -1$

(ii) (d) or $(1,2)$ **[1]**

Given equation of circle $2x^2 + 2y^2 - 4x - 8y - 45 = 0$

$\Rightarrow x^2 + y^2 + 2(-1)\,x + 2\,(-2)\,y - \frac{45}{2} = 0.$

$\therefore$ The centre of the circle is: $(1,2)$

(iii) (c) or 21 **[1]**

$\left(a^2 - 2\frac{b}{a^{-1}} + b^2\right)^{10} = (a^2 - 2ab + b^2)^{10} = (a-b)^{20}$

$\therefore$ Total number of terms $= 20 + 1 = 21$

(iv) (a) or $2x + y + 6 = 0$ **[1]**

As per question the point of intersection of the line and x-axis is $(-3,0)$.

Given, slope $= -2$.

Required equation of the line is: $y - 0 = -2(x+3) \Rightarrow 2x + y + 6 = 0$

(v) (c) or Both I and II **[1]**

The Cartesian product A × A has 16 elements among which are (0,2) and (1,3).

Hence 0,2,1,3 all are element of the set A.

i.e. A = {0,1,2,3}.

Also $(3,2) \in A \times A$

$\Rightarrow$ Both statements are true.

(vi) (d) or $\cos\theta$ decreases from -1 to 0 **[1]**

In III[rd] quadrant $\cos\theta$ **<u>decreases</u>** from -1 to 0

(vii) (a) or 93·32 **[1]**

Given, x_i: 2, 4, 5, 6, 8, 17 is 23·33.

Revised observations are:4, 8, 10, 12, 16, 34 i.e. $2x_i$

Hence, variance of revised observations = $2^2{\sigma_x}^2 = 4 \times 23 \cdot 33 = 93{\cdot}32$.

(viii) (a) or $\frac{5}{6}$ **[1]**

Given, the eccentricity of the ellipse with foci at (± 5, 0) and $x = \frac{36}{5}$ as one of the directrices.

$\Rightarrow ae = 5$ and $\frac{a}{e} = \frac{36}{5}$

$\Rightarrow a^2 = 36 \Rightarrow a = 6$

$\therefore e = \frac{5}{6}$

(ix) (c) or step-3 **[1]**

$y = sin^3x.cos^3x$

$= \frac{1}{8}sin^3 2x$

Differentiating w.r.t 'x' we get

$\Rightarrow \frac{dy}{dx} = \frac{3}{8}sin^2 2x \times \frac{d}{dx}(\sin 2x)$

$\Rightarrow \frac{dy}{dx} = \frac{3}{8}sin^2 2x. \cos 2x \times \frac{d}{dx}(2x)$ **(step-3)**

$\Rightarrow \frac{dy}{dx} = \frac{3}{4}sin^2 2x. \cos 2x.$

(x) (c) or Null set(φ) **[1]**

$(A - B) \cap B = (A \cap B') \cap B$

$= A \cap (B' \cap B)$ [Associative law]

$= A \cap \varphi$ [Laws of complementation]

$= \varphi$ [Boundedness law]

(xi) (b) or $x - 2 = 0$ **[1]**

Given, equation of parabola $y^2 = 4px$, where $p < 0$

and $p^2 + p - 2 = 0$

$\Rightarrow (p + 2)(p - 1) = 0$

$\Rightarrow p = -2$, ($\because p < 0$)

Required equation of directrix is: $x = -(-2) \Rightarrow x - 2 = 0$

(xii) (d) or The terms of S form an arithmetic progression with common difference 6. **[1]**

$$S_n = 3n^2 + 5n$$

Using, $T_n = S_n - S_{n-1} = 3n^2 + 5n - [3(n-1)^2 + 5(n-1)] = 6n + 2$

$$\therefore T_1 = 8, T_2 = 14, T_3 = 20, \ldots\ldots$$

Above is an A.P., having 1st term = 8 and common difference = 6

(xiii) (d) or 42 **[1]**

Total number of selections = $C(6,0) + C(6,1) + C(6,2) + C(6,3) = 42$.

(xiv) (c) or $(-\infty, 3)$ **[1]**

$\frac{-2}{x-3} > 0 \Rightarrow \frac{(x-3)}{(x-3)^2} < 0 \Rightarrow x - 3 < 0 \Rightarrow x < 3$ i.e. $x \in (-\infty, 3)$

(xv) (b) or Only (P) and (R) **[1]**

Given, circle $4x^2 + 4y^2 - 4ax - 4ay + a^2 = 0$

$\Rightarrow x^2 + y^2 - ax - ay + \frac{a^2}{4} = 0$

$$\Rightarrow x^2 + y^2 + 2\left(\frac{-a}{2}\right)x + 2\left(\frac{-a}{2}\right)y + \frac{a^2}{4} = 0$$

$$\Rightarrow \left[x^2 + 2\left(\frac{-a}{2}\right)x + \frac{a^2}{4}\right] + \left[y^2 + 2\left(\frac{-a}{2}\right)y + \frac{a^2}{4}\right] = \frac{a^2}{4}$$

$$\Rightarrow \left(x - \frac{a}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2$$

$\therefore$Centre$\left(\frac{a}{2}, \frac{a}{2}\right)$, Radius: $\frac{a}{2} \Rightarrow$ The circle touches both the axes.

Diameter = a.

$\left(\frac{a}{2}, \frac{a}{2}\right)$ satisfies $x + y = a$.

(xvi) (d) or A is false but R is true. **[1]**

Given set: {{0, {1}, 3}}

⇒ Cardinal number of the set = 1

⇒ Cardinal number of the power set of {{0, {1}, 3}} $= 2^1 = 2$

Hence A**ssertion (A)**: is false.

Reason (R): If A is a set with n(A) = m, the n (P(A)) = 2^m is true.

(xvii) (d) or A is false but R is true. **[1]**

Given, coordinate of the point P is (2, 4, 5).

⇒ coordinate of the point F is (2, 0, 5). [∵ F lies on ZX plane]

When we reflect a point (x, y, z) in YZ plane, we simply changing the sign of first variable.

∴ The image of the point F in YZ plane is $(-2,0,5)$.

Hence A**ssertion (A)**: is false.

When reflecting a point (x, y, z) in the ***XOY* plane**, the Z-coordinate changes its sign. The reflected point becomes $(x, y, -z)$.

Reason is true.

(xviii) Given, perpendicular distance of the line from origin is 4 units and the angle which is normal makes with positive direction of x-axis is 15°. **[1]**

Using, normal form of the equation of line we get,

$x \cos 15° + y \sin 15° = 4$

⇒ Intercepts are: $\frac{4}{\cos 15°}$ and $\frac{4}{\sin 15°}$

∴ Sum of the intercepts $= \frac{4}{\cos 15°} + \frac{4}{\sin 15°} = 4\left[\frac{2\sqrt{2}}{\sqrt{3}-1} + \frac{2\sqrt{2}}{\sqrt{3}+1}\right] = 8\sqrt{2}\left[\frac{\sqrt{3}+1+\sqrt{3}-1}{2}\right] = 8\sqrt{6}$

(xix) Given, word '**LAHORE**', also L and A are always together. **[1]**

(LA) HORE

∴ Total number of words = $5! \times 2! = 240$

(xx) $f(x) = \pi + \sin^2 x$ **[1]**

We know, $-1 \leq \sin x \leq 1$

$\Rightarrow 0 \leq sin^2 x \leq 1$

$\Rightarrow \pi \leq \pi + \sin^2 x \leq \pi + 1$

$\therefore$ Range of the function: $[\pi, \pi + 1]$

SECTION B – 14 MARKS

Question 2 **[2]**

(i) The starting point Q is the origin (0,0).
Point P is at a horizontal distance of 10 meters from Q i.e. $x = 10$
Vertical height of 2 meters i.e. $y = 2$.
The ramp connects Q$(0, 0)$ and P$(10, 2)$.
The equation of the line passing through (0,0) and (10,2):
The equation of the ramp is $y = \frac{x}{5}$ or $x - 5y = 0$.
The angle of inclination θ is the angle the ramp makes with the horizontal (x-axis).

$$tan\theta = m = \frac{1}{5},\ \theta = \tan^{-1}(\frac{1}{5}) \approx (11.31)^{\circ}.$$

Question 3 **[2]**

Given, a, b and $c \Rightarrow b^2 = ac$

Given, quadratic equation: $ax^2 + bx + c = 0$

(i) Discriminant (D) = $b^2 - 4ac = ac - 4ac = -3ac < 0$, since $a, c > 0$.
Hence equation has imaginary roots.

(ii) Product of the roots = $\frac{c}{a} = \frac{ac}{a^2} = \frac{b^2}{a^2}$

Question 4 **[2]**

$f(x) = \frac{xsinx}{1+cosx} = x \times \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2cos^2\frac{x}{2}} = x\tan\frac{x}{2}$

Differentiating w.r.t. 'x' we get

$f^{/}(x) = \frac{d}{dx}\left(x\tan\frac{x}{2}\right)$

$= \tan\frac{x}{2} + \frac{x}{2}sec^2\frac{x}{2}$

$\therefore f'\left(\frac{\pi}{2}\right) = \tan\frac{\pi}{4} + \frac{\pi}{4}sec^2\frac{\pi}{4}$

$= 1 + \frac{\pi}{2}$

Question 5 **[2]**

(i) Given, the sum of binomial coefficients in the expansion in the expansion of $(x+y)^n$ is 256.

$\Rightarrow 2^n = 256$

$\Rightarrow 2^n = 2^8 \Rightarrow n = 8.$

$\therefore$ Total number of terms $= 8 + 1 = 9$

We know that the coefficient of middle most term is the greatest.

Hence $\left(\frac{9+1}{2}\right)^{\text{th}}$. i.e. 5^{th}. term coefficient is the greatest.

OR

(ii) Given, binomial expansion is $(1+x)^n$

$T_{r-5} = T_{1+(r-6)} = C(34, r-6)x^{r-6}$

$T_{2r-1} = T_{1+(2r-2)} = C(34, 2r-2)x^{2r-2}$

As per question, $C(34, r-6) = C(34, 2r-2)$

$\Rightarrow 34 = r - 6 + 2r - 2$ [since, $C(n,a) = c(n,b) \Rightarrow a + b = n$]

$\Rightarrow 3r = 42$

$\Rightarrow r = 14$

$\therefore$ Value of $r = 14.$

Question 6 **[2]**

(i) **L.H.L.** $= \underset{x \to 0^-}{lt} f(x) = \underset{x \to 0^-}{lt} \frac{|sinx|}{x}$

$= \underset{x \to 0^-}{lt} \frac{(sinx)}{x} = -1$

R.H.L. $= \underset{x \to 0^+}{lt} \frac{|sinx|}{x} = \underset{x \to 0^+}{lt} \frac{sinx}{x} = 1$

$\therefore L.H.L. \neq R.H.L.$

$\therefore$ limit does not exist

OR

(ii)

$$= \underset{x \to \pi}{lt} \frac{1 - Sin\frac{x}{2}}{Cos\frac{x}{2}\left[Cos\frac{x}{4} - Sin\frac{x}{4}\right]}$$

$$= \underset{x \to \pi}{lt} \frac{\left(Cos\frac{x}{4} - Sin\frac{x}{4}\right)^2}{\left(Cos^2\frac{x}{4} - Sin^2\frac{x}{4}\right)\left(Cos\frac{x}{4} - Sin\frac{x}{4}\right)} = \underset{x \to \pi}{lt} \frac{Cos\frac{x}{4} - Sin\frac{x}{4}}{Cos^2\frac{x}{4} - Sin^2\frac{x}{4}}$$

$$= \underset{x \to \pi}{lt} \frac{1}{\left(Cos\frac{x}{4} + Sin\frac{x}{4}\right)}$$

By applying limit, we get

$$= \frac{1}{Cos\frac{\pi}{4} + Sin\frac{\pi}{4}} = \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \frac{1}{\frac{2}{\sqrt{2}}}$$

$= \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ **Ans.**

Question 7 **[2]**

(i) The number of subsets of a given set having m elements=2^m

As per question $2^m - 2^n = 112$

$\Rightarrow 2^n (2^{m-n} - 1) = 112 \Rightarrow 2^n (2^{m-n} - 1) = 2^4 \times 7$

On comparing we get

$\Rightarrow n=4$ and $2^{m-n} - 1 = 7$

$\Rightarrow 2^{m-n} = 8 = 2^3$

$\Rightarrow m - n = 3$

$\Rightarrow m - 4 = 3$

$\Rightarrow m = 7$

Ans. m=7, n=4

OR

(ii) $n(A' \cap B') = n(A \cup B)'$

$= n(\cup) - n(A \cup B)$

$= n(\cup) - [n(A) + n(B) - n(A \cap B)]$

$= n(\cup) - [29 + 240 - 110]$

$= n(\cup) - [530 - 110]$

$= 700 - 420$

$= \mathbf{280}$ **Ans.**

Question 8 **[2]**

There are 4 kings in a deck of 52 cards,

$\Rightarrow$ The probability 1st card drawn was a king= $\frac{4}{52} = \frac{1}{13}$.

If the first card drawn was a king, there are now 3 kings left in the remaining 51 cards.

$\Rightarrow$ The probability 2nd. card drawn was a king $= \frac{3}{51} = \frac{1}{17}$

$\therefore$Probability of Both Cards being Kings = $= \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$

SECTION C – 21 MARKS

Question 9 **[4]**

(i) Distance between two points: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Given $A\ (1,2,3), B\ (0,4,1), C(-1,-1,-3)$

Length $AB\ = 3$ and $AC = 7$

(ii) Coordinates of the point where the bisector of ∠BAC meets BC:

In triangle ABC, the bisector of ∠BAC meets BC at a point, say D. According to the Angle Bisector Theorem, D divides BC in the ratio of $AB : AC$.

So, the point D divides the line segment BC internally in the ratio 3 : 7.

Applying section formula: $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

$B\ (0,4,1), C(-1,-1,-3)$The Point D divides BC internally in the ratio $3:7$

Therefore, the coordinates of the point where the bisector of ∠BAC meets BC are $\left(\frac{-3}{10}, \frac{5}{2}, \frac{-1}{5}\right)$.

Question 10 **[3]**

Given that α and β are the roots of $qx^2 - px + r = 0$

$\therefore\ \alpha + \beta = \frac{p}{q}, \alpha.\beta = \frac{r}{q}$

New roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$

S = $\alpha + \frac{1}{\beta} + \beta + \frac{1}{\alpha} = \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} = \alpha + \beta + \frac{\alpha+\beta}{\alpha\beta} = \frac{p}{q} + \frac{p}{r} = p\left(\frac{q+r}{qr}\right)$

P = $\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\alpha\beta} + 2 = \frac{r}{q} + \frac{q}{r} + 2 = \frac{(q+r)^2}{qr}$

Requires quadratic equation: $x^2 - Sx + P = 0$

$\Rightarrow x^2 - p\left(\frac{q+r}{qr}\right)x + \frac{(q+r)^2}{qr} = 0$

Question 11 **[3]**

Method I:

Substitute the equation of the line into the equation of the circle:

$y = x + 1$ in the equation of circle$x^2 + y^2 = 5$

$$x^2 + (x+1)^2 = 5$$

$$x^2 + (x^2 + 2x + 1) = 5$$

$$2x^2 + 2x - 4 = 0 \Longrightarrow x^2 + x - 2 = 0$$

Factor the quadratic equation: $(x+2)(x-1) = 0$

$$\Longrightarrow x = -2\ or\ x = 1.$$

Substitute these x values back into the equation of the line $y = x + 1$:

If $x = -2$, then $y = -2 + 1 = -1$. The point is $(-2,-1)$.

If $x = 1$, then $y = 1 + 1 = 2$. The point is $(1,2)$.

The line intersects the circle at the points $(-2,-1)$ and $(1,2)$.

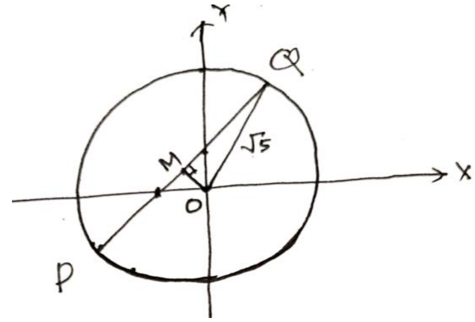
Length of the chord = $\sqrt{(1+2)^2 + (2+1)^2} = \sqrt{18} = 3\sqrt{2}$ Units

Method II

Perpendicular distance from centre of a circle $(0, 0)$ to the chord: $x - y + 1 = 0$ is $\frac{1}{\sqrt{2}}$

Radius of circle is $\sqrt{5}$ units

Based on the theorem a perpendicular drawn from centre of a circle to chord divides it into two equal halves.

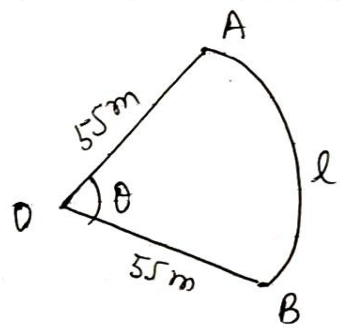


$$\therefore \text{Half of the length of the chord} = \sqrt{5 - \frac{1}{2}} = \sqrt{\frac{9}{2}}$$

$$= \frac{3}{\sqrt{2}}$$

Length of the chord: $2 \times \frac{3}{\sqrt{2}} = 3\sqrt{2}$ Units

Question 12 **[3]**



$Let\ Arc\ AB = lm$

$\therefore 55 + 55 + l = 176$

$l = 176 - 110$

$l = 66\ m$

$Let\ \angle AOB = \theta^c$

$l = r\theta^c$

$66 = 55\theta$

$\therefore \theta^c = \frac{66}{55} = \frac{6}{5}$

Area of the sector $= \frac{1}{2}r^2\theta^c = \frac{1}{2} \times 55^2 \times \frac{6}{5} = 55 \times 33 = 1815m^2$

Question 13 **[3]**

(i) Given, one root is $2 - i\sqrt{3}$ i.e. imaginary root.

$\therefore$ Other, root is $2 + i\sqrt{3}$

S = $2 - i\sqrt{3} + 2 + i\sqrt{3} = 4$

P = $\left(2 - i\sqrt{3}\right)\left(2 + i\sqrt{3}\right) = 4 + 3 = 7$

Requires quadratic equation: $x^2 - Sx + P = 0$

$$\Rightarrow x^2 - 4x + 7 = 0$$

$$\Rightarrow x^2 + (-4)x + 7 = 0$$

On comparing we get, $a = -4, b = 7$.

$$\therefore a + b = -4 + 7 = 3.$$

OR

(ii) Let, z = $x + iy\ , x\ , y \in$ R.

$$\therefore \frac{z-1}{z+1} = \frac{(x-1)+iy}{(x+1)+iy} = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy} = \frac{\left(x^2-1+y^2\right) + i\left(y(x+1)-y(x-1)\right)}{(x+1)^2+y^2}$$

Since, $\frac{z-1}{z+1}$ is purely imaginary $\Rightarrow Re\left(\frac{z-1}{z+1}\right) = 0$

$$\Rightarrow \frac{(x^2 - 1 + y^2)}{(x+1)^2 + y^2} = 0$$

$$\Rightarrow x^2 - 1 + y^2 = 0$$

$$\Rightarrow x^2 + y^2 = 1 \Rightarrow |z|^2 = 1 \Rightarrow |z| = 1$$

Question 14 **[3]**

(i) (a) $\frac{dy}{dx} = \frac{d}{dx}(1 - x^3 - 6x^2 - 9x)$

$= 0 - 3x^2 - 12x - 9$

$\frac{dy}{dx} = -3x^2 - 12x - 9$ *Ans*

(b) Since the tangents to the curve at points B and C are parallel to the *x-axis*:

$\Rightarrow \frac{dy}{dx} = 0$

$\Rightarrow -3x^2 - 12x - 9 = 0$

$\Rightarrow x^2 + 4x + 3 = 0$

$\Rightarrow (x+3)(x+1) = 0$

$\Rightarrow x = -3, -1$

$\therefore y = 1 - x^3 - 6x^2 - 9x$

When, $x = -3 \Rightarrow y = 1 - (-3)^3 - 6(-3)^2 - 9(-3) = 1 + 27 - 54 + 27 = 1$

When, $x = -1 \Rightarrow y = 1 - (-1)^3 - 6(-1)^2 - 9(-1) = 5$.

Hence, coordinates of B(−3,1), C (−1,5).

OR

(ii) $Let f(x) = \sqrt{3x+4}$

$f(x+h) = \sqrt{3(x+h)+4}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{3(x+h)+4} - \sqrt{3x+4}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{3(x+h)+4} - \sqrt{3x+4}}{-h} \times \frac{\sqrt{3(x+h)+4} + \sqrt{3x+4}}{\sqrt{3(x+h)+4} + \sqrt{3x+4}}$$

$$= \lim_{h \to 0} \frac{3(x+h)+4-3x-4}{h[\sqrt{3x+3h+4} + \sqrt{3x+4}]}$$

$$= \lim_{h \to 0} \frac{3}{[\sqrt{3x+3h+4} + \sqrt{3x+4}]}$$

$$= \frac{3}{\sqrt{3x+4} + \sqrt{3x+4}}$$

$\frac{dy}{dx} = \frac{3}{2\sqrt{3x+4}}$ **Ans.**

Question 15 **[3]**

(i) Let $R\cos\alpha = 115$ and $R\sin\alpha = 115\sqrt{3}$

$\therefore R^2 = (115)^2 + (115\sqrt{3})^2$

$\Rightarrow R^2 = 52900$

$\Rightarrow R = 230$

$\therefore R\cos\alpha = 115,\ R\sin\alpha = 115\sqrt{3}$

$\therefore \tan\alpha = \frac{115\sqrt{3}}{115} = \sqrt{3}$

$\therefore \alpha = \frac{\pi}{3}$

$\therefore V = R\cos\alpha \, Sin\omega t + R Sin\alpha \cos\omega t$

$= R(\sin\omega t. \cos\alpha + \cos\omega t \sin\alpha)$

$V = R Sin(\omega t + \alpha)$

$V = 230 \sin(\omega t + \frac{\pi}{3})$ **Ans.**

Method -II

$V = 115\, Sin\omega t + 115\sqrt{3}\cos\omega t$

$= 115 \times 2 \left[Sin\omega t \times \frac{115}{\mathbf{115 \times 2}} + \cos\omega t \times \frac{115\sqrt{3}}{\mathbf{115 \times 2}}\right], \because \sqrt{(115)^2 + (115\sqrt{3})^2} = 115 \times 2$

$= \mathbf{230} \left[Sin\omega t \times \frac{\mathbf{1}}{\mathbf{2}} + \cos\omega t \times \frac{\sqrt{3}}{\mathbf{2}}\right]$

$= \mathbf{230} \left[Sin\omega t \times \cos\frac{\pi}{3} + \cos\omega t \times \sin\frac{\pi}{3}\right]$

$= \mathbf{230} \sin\left(\omega t + \frac{\pi}{3}\right)$ **Ans.**

OR

(ii) (a) Given, $A + B = \frac{\pi}{4}$

$\Rightarrow \cot(A + B) = 1$

$\Rightarrow \frac{\cot A . \cot B - 1}{\cot B + \cot A} = 1$

$\Rightarrow \cot A . \cot B - 1 = \cot B + \cot A$

$\Rightarrow \cot A . \cot B - \cot B - \cot A + 1 = 1 + 1$

$\Rightarrow \cot A (\cot B - 1) - (\cot B - 1) = 2$

$\Rightarrow (cotA - 1)(cotB - 1) = 2$

(b) $\because \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$

$\therefore \left(cot\frac{\pi}{8} - 1\right)\left(cot\frac{\pi}{8} - 1\right) = 2$

$\therefore \left(Cot\frac{\pi}{8} - 1\right)^2 = 2$

$\therefore cot\ \frac{\pi}{8} - 1 = \pm\sqrt{2}$

$\therefore cot\ \frac{\pi}{8} - 1 = \pm\sqrt{2} + 1$

$\Rightarrow cot\frac{\pi}{8} = \sqrt{2} + 1$ $\left(\boldsymbol{as\frac{\pi}{8} = actue\ angle\ \therefore cot\frac{\pi}{8} = +ve}\right)$ ***Ans.***

SECTION D – 25 MARKS

Question 16 **[5]**

(i) For the parabola: $y^2 = 8x$, $4a = 8$, so $a = 2$. The focus is $S(2,0)$.

The equation of a line passing through the focus $S(2,0)$ with a slope

$m = tan(60^0) = \sqrt{3}$

$y - 0 = \sqrt{3}(x - 2) \Rightarrow y = \sqrt{3}x - 2\sqrt{3}$.

To find the points of intersection with the parabola, substitute y in the equation of the parabola: $\left(\sqrt{3}x - 2\sqrt{3}\right)^2 = 8x$

$3x^2 - 12x + 12 = 8x$

$3x^2 - 20x + 12 = 0$

Let the roots of this quadratic be $x_1\ and\ x_2$.

The corresponding $y -$ coordinates are $y_1 = \sqrt{3}x_1 - 2\sqrt{3}$ and $y_2 = \sqrt{3}x_2 - 2\sqrt{3}$.

The endpoints of the focal chord are $(x_1, y_1)\ and\ (x_2, y_2)$.

The length of the focal chord = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x_2 - x_1)^2 + 3(x_2 - x_1)^2}$

$= 2(x_2 - x_1)$

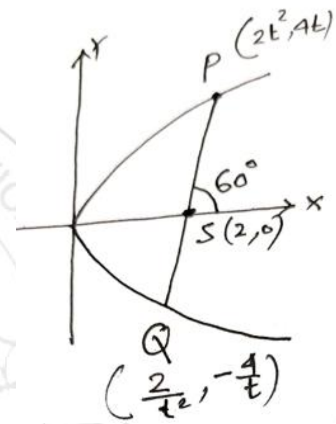
From the quadratic equation $3x^2 - 20x + 12 = 0$

Sum of roots $x_1 + x_2 = \frac{20}{3}$ and Product of roots $x_1 \cdot x_2 = \frac{12}{3} = 4$

$$(x_2 - x_1)^2 = (x_1 + x_2)^2 - 4x_1 \cdot x_2$$
$$= \frac{400}{9} - 16 = \frac{256}{9}$$
$$(x_2 - x_1) = \frac{16}{3}$$

The length of the focal chord is $2 \times \frac{16}{3} = \frac{32}{3}$ units.

Method -II



Give parabola: $y^2 = 8x$, $4a = 8$, so $a = 2$. The focus is $S(2{,}0)$.

End points of focal chord are $P(2t^2, 4t)$ and $Q\left(\frac{2}{t^2}, -\frac{4}{t}\right)$, where t is the parameter.

$\because$ the focal chord making angle 60° with x- axis, slope of SP = $\tan 60° = \sqrt{3}$.

$\Rightarrow \sqrt{3} = \frac{4t}{2t^2 - 2} \Rightarrow \sqrt{3}t^2 - 2t - \sqrt{3} = 0 \Rightarrow t = \sqrt{3}$ or $-\frac{1}{\sqrt{3}}$ (Neglected, since P lies in 1st quadrant)

$\therefore$ P$\left(6, 4\sqrt{3}\right)$ and Q$\left(\frac{2}{3}, -\frac{4}{\sqrt{3}}\right)$

$\therefore$ PQ = $\sqrt{\left(6 - \frac{2}{3}\right)^2 + \left(4\sqrt{3} + \frac{4}{\sqrt{3}}\right)^2} = \sqrt{\frac{16^2}{3^3} + 16 \times \frac{16}{3}} = \sqrt{\frac{16^2}{3^3}\left(1 + \frac{1}{3}\right)} = \frac{16}{3} \times \frac{2}{3} = \frac{32}{3}$

OR

(ii) Satellite(S) moves so that the sum of its distances from the points $f_2\ (-2, 0)$ and $f_1\ (2, 0)$ is 8 units. i.e. $Sf_1 + Sf_2 = 8$.

Using focal property of ellipse, we get $2a = 8$.

$\therefore$ Length of major axis is 8.

(a) $\therefore$Length of semi-major axis a is 4 units.
Since, foci $f_2\ (-2{,}0)$ and $f_1\ (2{,}0) \Rightarrow$ Major axis is along $x -$ axis.

Also centre is $(0{,}0)$.

Hence required equation of ellipse is : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore ae = 2$

(b) $\Rightarrow e = \frac{2}{4} = \frac{1}{2}$.
Using, $b^2 = a^2(1 - e^2)$, we get $b^2 = 12$.

(c) ∴ Required equation of elliptical orbit is $\frac{x^2}{16}+\frac{y^2}{12}=1$

Using, distance between focus and corresponding vertex = $a - ae$,

(d) ∴ $f_1P = 4-2 = 2$ units.

Question 17 **[5]**

(i) (a) Given word: **DEVASTATION**

AATT D E V S I O N

There are all together 9 distinct letters of which 2 pair of alike and 7 distinct objects.

Resultant word is a 4-letter word.

Case-1 Selection of 4 distinct letters from 9 distinct letters namely: A T D E V S I O N

Number of selections = C (9,4) = $\frac{9\times8\times7\times6\times5}{4!}$ = 126.

Case – II Selection of any one pair of alike from [**AA**, **TT**], 2 distinct letters from 8 distinct letters.

Number of selections = C (2,1) × C (8,2) = 2 × $\frac{8\times7}{2!}$ = 56

Case – III Selection of two pair of alike from [**AA**, **TT**]

Number of selections = C (2,2) =1

∴ Total number of selections = $126+56+1 = 183$

(b) Total number of permutations (words) = $126\times4!+56\times\frac{4!}{2!}+1\times\frac{4!}{2!\times2!}$

$= 126\times 24+56\times12+1\times6 = 702.$

OR

(ii) Given word is **CRICKET**, which is 7 letters word having 5 distinct letters **EIKR T** and 2 are alike **CC**.

(a) Total number of words = $\frac{7!}{2!}=\frac{7\times720}{2}=7\times360=2520.$ **[1]**

(b) As per dictionary order: **[4]**

Number of words of type CC * * * * * = 5! = 120

Number of words of type CE * * * * * = 5! = 120

Number of words of type CI * * * * * = 5! = 120

Number of words of type CK * * * * * = 5! = 120

Number of words of type CRC * * * * = 4! = 24

Number of words of type CRE * * * * = 4! = 24

Number of words of type CRICE * * = 2! = 2

Total = 530

As per alphabetical order the next word is CRICKET.

Hence the rank of the word CRICKET is 531.

Question 18 **[5]**

(i)

$$\cos\alpha + \left\{\cos\left(\frac{4\pi}{3}+\alpha\right) + \cos\left(\frac{2\pi}{3}+\alpha\right)\right\} = \cos\alpha + 2\,.\cos\left(\frac{2\pi+2\alpha}{2}\right)\cos\frac{\frac{2\pi}{3}}{2}$$

$$= \cos\alpha + 2\ \cos\left(\frac{\pi+\alpha}{2}\right)\cos\frac{\pi}{3}$$

$$= \cos\alpha - 2\,.\cos\alpha.\frac{1}{2} = 0$$

Let $x = \cos\alpha$, $y = \cos\left(\frac{2\pi}{3}+\alpha\right)$ and $z = \cos\left(\frac{4\pi}{3}+\alpha\right)$

We know if, $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

$\therefore cos^3\alpha + cos^3\left(\alpha+\frac{2\pi}{3}\right) + cos^3\left(\alpha+\frac{4\pi}{3}\right) = 3\cos\alpha\cos\left(\alpha+\frac{2\pi}{3}\right)\cos\left(\alpha+\frac{4\pi}{3}\right)$.

Now $3\cos\alpha\cos\left(\alpha+\frac{2\pi}{3}\right)\cos\left(\alpha+\frac{4\pi}{3}\right) = \frac{3}{2}\cos\alpha\left\{2\cos\left(\frac{4\pi}{3}+\alpha\right)\cos\left(\frac{2\pi}{3}+\alpha\right)\right\}$

$$= \frac{3}{2}\cos\alpha\left\{\cos\frac{2\pi}{3} + \cos(2\pi+2\alpha)\right\}$$

$$= \frac{3}{2}\cos\alpha\left\{-\frac{1}{2} + \cos 2\alpha\right\}$$

$$= \frac{3}{2}\cos\alpha\left\{-\frac{1}{2} + 2cos^2\alpha - 1\right\}$$

$$= \frac{3}{4}\cos\alpha\,\{4cos^2\alpha - 3\}$$

$$= \frac{3}{4}\cos\alpha\,\{4cos^3\alpha - 3\cos\alpha\}$$

$$= \frac{3}{4}\cos 3\alpha$$

$\therefore k = 3$

OR

(ii) The function f is defined by $f(x) = x^2 - 6x + 17$.

$$y = x^2 - 6x + 17$$

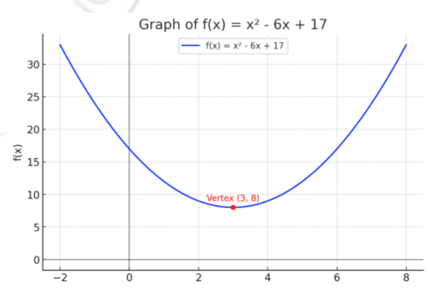
$\Rightarrow x^2 - 6x = y - 17$

$\Rightarrow x^2 - 6x + 9 = y - 17 + 9$

$\Rightarrow (x-3)^2 = (y-8)$

which is an upward parabola with vertex (3,8).

Hence the graph of the function is as drawn alongside.



Method -I

(i) From graph y −coordinate of the vertex is 8. ⇒ Minimum value of $f(x)$ =8.

(ii) Since the **minimum value** of $f(x)$ is 8 and the parabola opens upwards, $f(x) > 0\ \forall x \in$ R. Hence function is **always positive**.

(iii) Since the **minimum value** is 8 and it goes up to +∞, the range is: [8, +∞).

Method -II

$f(x) = x^2 - 6x + 17$

$D = 36 - 4 \times 1 \times 17 = -32 < 0, a = 1 > 0 \Rightarrow f(x)$ has minimum value.

(i) Minimum value = $-\frac{D}{4a} = -\frac{(-32)}{4} = 8$.

(ii) $\because D < 0 \Rightarrow f(x)$ has same sign that of a. Here $a = 1 > 0 f(x) > 0 \;\forall x \in$ R.
Hence function is **always positive**.

(iii) $(x) = x^2 - 6x + 17 = 8 + (x-3)^2 \Rightarrow$ the range is: [8, +∞).

Question 19 **[5]**

CI	x	f	d = x − A	fd	fd^2
20-25	22.5	170	−15	−2550	38250
25-30	27.5	110	−10	−1100	11000
30-35	32.5	80	−5	−400	2000
35-40	37.5	45	0	0	0
40-45	42.5	40	5	200	1000
45-50	47.5	30	10	300	3000
50-55	52.5	25	15	375	5625
		500		**−3175**	**60875**

Assumed mean A = 37·5

Arithmetic Mean $= AM + \frac{\Sigma fd}{N}$

$= 37 \cdot 5 - 6 \cdot 35 = 31 \cdot 15$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = \sqrt{\frac{60875}{500} - \left(\frac{-3175}{500}\right)^2} = 9 \cdot 02$$

$\bar{x} - \sigma = 31 \cdot 15 - 9 \cdot 02 =$ 22·13

$\bar{x} + \sigma = 31 \cdot 15 + 9 \cdot 02 = 40 \cdot 17$.

the number of employees whose age is between 22·13 and 40·17

From the table, we can see that this age range includes the intervals:

- 20-25 (170 employees)
- 25-30 (110 employees)
- 30-35 (80 employees)
- 35-40 (45 employees)

Total employees in this range = 170 + 110 + 80 + 45 = 405

Percentage of employees = $\left(\frac{Employees\ in\ range}{Total\ employees}\right)100 = \frac{405}{500} \times 100 = 81\%$

Answers:

- Mean = 31·15
- Standard Deviation = 9·02
- Percentage of employees whose age lies between $\bar{x} - \sigma$ and $\bar{x} + \sigma = 81\%$

Question 20 **[5]**

	1st. G.P.	2nd G.P.	3rd. G.P.	.. ..	nth G.P.
1st Term	1	$2^2 - 1$	$2^3 - 1$		$2^n - 1$
Common Ratio	$\frac{1}{2}$	$\frac{1}{2^2}$	$\frac{1}{2^3}$		$\frac{1}{2^n}$
S_∞	$S_1 = \frac{1}{1 - \frac{1}{2}}$ = 2	$S_2 = \frac{2^2 - 1}{1 - \frac{1}{2^2}}$ = 2^2	$S_3 = \frac{2^3 - 1}{1 - \frac{1}{2^3}}$ = 2^3		$S_n = \frac{2^n - 1}{1 - \frac{1}{2^n}}$ = 2^n

$\therefore S_1 + S_2 + S_3 + \ldots + S_n = 2 + 2^2 + 2^3 + \ldots\ldots\ldots + 2^n$

$= \frac{2(2^n - 1)}{2 - 1} = 2(2^n - 1)$