APPLIED MATHEMATICS

Maximum Marks: 80
Time Allotted: Three Hours
Reading Time: Additional Fifteen Minutes

Instructions to Candidates

1. You are allowed an **additional fifteen minutes** for **only** reading the paper.
2. You must **NOT** start writing during reading time.
3. The question paper has **14 printed pages**.
4. It consists of **20 questions and four sections: A, B, C and D**. **All questions are compulsory.**
5. **Section A** comprises **very short answer questions** of **1 mark** each.
6. **Section B** consists of **short answer questions** of **2 marks** each.
7. **Section C** consists of **moderately long answer questions** of **3 marks** each.
8. **Section D** consists of **long answer questions** of **5 marks** each.
9. Internal choices have been provided in **three** questions, each in **Sections B, C and D.**
10. While attempting **Multiple Choice Questions in Section A**, you are required to **write only ONE option as the answer.**
11. The intended marks for questions or parts of questions are given in the brackets [].
12. All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
13. Mathematical tables and graph papers are provided.

Instruction to Supervising Examiner

Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Please note that Specimen Question Paper in the subject provides a realistic format of the Board Question Paper and should be used as a practice tool. However, the Board Question Paper may not necessarily replicate the Specimen Question Paper as individual questions may vary. The weightage allocated to various topics, as given in the syllabus, will be strictly adhered to.

SECTION A – 20 MARKS

Question 1

In subparts (i) to (xvii) choose the correct options and in subparts (xviii) to (xx), answer the questions as instructed.

(i) A relation R is defined on the set of students of a class such that $R = \{(a, b)$: student a and student b have at least one common friend$\}$ **[1]**

Which of the following statements is correct? **(Understand)**

(a) R is an equivalence relation.

(b) R is neither symmetric nor transitive.

(c) R is symmetric but not necessarily transitive.

(d) R is reflexive and transitive but not symmetric.

(ii) A bank offers two investment schemes: **[1]**

- Scheme A: 10% compounded annually
- Scheme B: 10% compounded quarterly

Which scheme gives a higher effective annual return? **(Understand)**

(a) Scheme A

(b) Scheme B

(c) Cannot be determined

(d) Both give the same return

(iii) If $A = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$ is both symmetric and skew-symmetric, then $x + y$ is: **(Apply)** **[1]**

(a) 0

(b) 3

(c) -3

(d) 6

(iv) The function $y = -x^3 + ax^2 + 2x - 27$ attains its maximum at $x = 1$. Then the value of a is: **[1]**

(Apply)

(a) $\frac{-1}{2}$

(b) $\frac{1}{2}$

(c) 2

(d) -2

(v) Trials of a random experiment are called Bernoulli trials if: **[1]**

(P) The number of trials is finite.

(Q) The trials are dependent on each other.

(R) The probability of success or failure remains the same in each trial.

Choose the correct answer. **(Recall)**

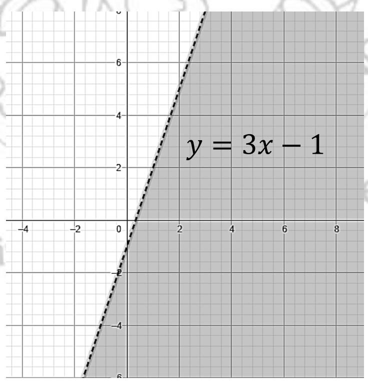
(a) Only (P) and (Q)

(b) Only (Q) and (R)

(c) Only (P) and (R)

(d) (P), (Q) and (R)

(vi) **[1]**



The open half plane shaded in the above graph formed by the line $y = 3x - 1$ is represented as: **(Understand)**

(a) $3x - y \geq 1$

(b) $3x - y \leq 1$

(c) $3x - y > 1$

(d) $3x - y < 1$

(vii) Find the present value of perpetuity of ₹1000 payable at the end of each year, if money is worth 5% per annum. **(Apply)** **[1]**

(a) ₹18,000

(b) ₹28,000

(c) ₹10,000

(d) ₹20,000

(viii) The production rate of a manufacturing company is modelled by the function: $P(t) = 500 + 20t$, where $P(t)$ denotes the number of units produced per day after t days. **[1]**

Assertion: The rate of change of production with respect to time is constant.

Reason: The production function is a linear function of t.

Choose the correct answer from the following options. **(Understand)**

(a) Both Assertion and Reason are true, and Reason is the correct explanation for Assertion

(b) Both Assertion and Reason are true, but Reason is not the correct explanation for Assertion

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(ix) Which of the following is an advantage of weighted index numbers in real life situations? **(Recall)** **[1]**

(a) Base year is not needed.

(b) All items are treated equally.

(c) Calculations are always simple.

(d) Commodities are assigned weights according to their relative importance.

(x) Let A and B be square matrices of the same order. **[1]**

Statement I: If $AB = BA$, then $(A + B)^2 = A^2 + 2AB + B^2$.
Statement II: If $AB \neq BA$, then $(A + B)^2 = A^2 + B^2$.

(Understand)

(a) Statement I is true and Statement II is false.

(b) Statements I is false and Statement II is true.

(c) Both the statements are true.

(d) Both the statements are false.

(xi) If the mean and variance of a binomial distribution are 2 and $\frac{4}{3}$ respectively, then the number of trials, n is: **[1]**

(Apply)

(a) 6

(b) $\frac{1}{6}$

(c) $\frac{5}{6}$

(d) $\frac{5}{36}$

(xii) Observe the graphs given below: **[1]**

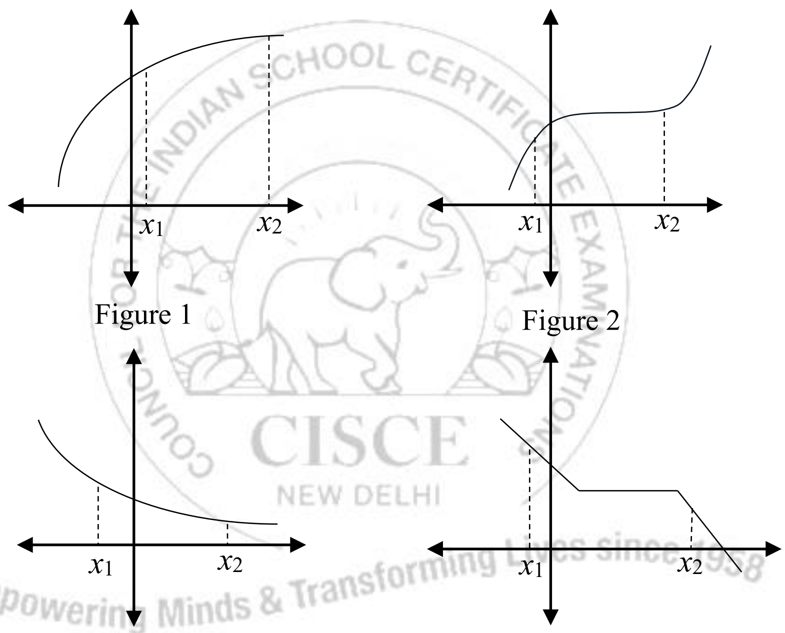


Figure 1 Figure 2

Figure 3 Figure 4

Statement I: Figure 1 represents an increasing function and Figure 4 represents a strictly decreasing function in the defined interval.

Statement II: Figure 2 represents a strictly increasing function and Figure 3 represents a decreasing function in the defined interval.

(Understand)

(a) Statement I is true and Statement II is false.

(b) Statements I is false and Statement II is true.

(c) Both the statements are true.

(d) Both the statements are false.

(xiii) If the feasible region of a Linear Programming Problem (LPP) is unbounded, then the maximum or minimum value of the objective function: **(Recall)** **[1]**

(a) always exists.

(b) never exists.

(c) may or may not exist.

(d) can never be determined as the feasible region has no corner point.

(xiv) In which EMI method, does the interest decrease with every installment paid? **(Recall)** **[1]**

(a) Flat rate method

(b) Discount method

(c) Simple interest method

(d) Reducing balance method

(xv) **Assertion:** The mean of a probability distribution is always non-negative. **[1]**

Reason: Probability of an event is always non-negative.

(Understand)

(a) Both Assertion and Reason are true, and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true, but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(xvi) The system of equation $3x - 2y = 4$ and $6x - 4y = 9$ is: **(Understand)** **[1]**

(a) consistent

(b) dependent

(c) inconsistent

(d) homogeneous

(xvii) If the rate of return on an investment increases, then the time taken for the investment to double will generally: **(Understand)** **[1]**

(a) increase

(b) decrease

(c) become zero

(d) remain unchanged

(xviii) Find the degree of the differential equation $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2} = x + y$. **(Recall)**

(xix) In a batch of 25 students at a dance academy, there are 12 boys and 13 girls. Among them, 5 boys and 7 girls are members of the Indian classical dance team. A student is selected at random to lead the team at an international dance festival. If it is known that the selected student is from the Indian classical dance team, find the probability that the student is a girl. **(Apply)** **[1]**

(xx) Evaluate $\int e^x(cotx + logsinx)dx$ **(Evaluate)** **[1]**

SECTION B – 14 MARKS

Question 2

A landowner owns a triangular plot of land with vertices: $A(10,20), B(40,25), C(30,60)$. The owner plans to utilise the plot for starting a new business based on its area.

- If the area of the plot is greater than 100 square units, the owner wants to construct a grocery shop.
- Otherwise, the owner wants to open a tea shop.

Based on the above information, answer the following questions:

(i) Using determinants, find the area of the triangular plot. **(Analyse)** **[1]**

(ii) Determine the type of shop the owner should construct. **(Analyse)** **[1]**

Question 3 **[2]**

(i) Using properties of definite integral, evaluate: $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}} dx$ **(Evaluate)**

OR

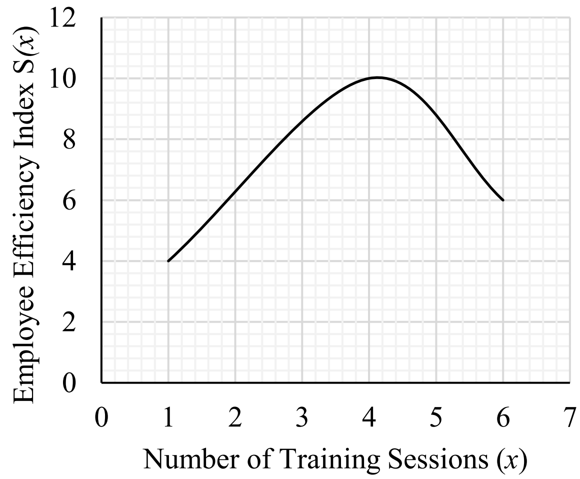
(ii) Evaluate: $\int_{-2}^{3}|x-1|\,dx$ **(Evaluate)**

Question 4 **[2]**

A group of friends is bowling at an entertainment mall. Each player gets 10 attempts. If the probability of a specific player hitting the pins on any given throw is $\frac{5}{6}$, what is the probability that he will hit the pins at least 9 times? **(Apply)**

Question 5

(i) The graph below shows the Employee Efficiency Index $S(x)$ of a company with respect to the number of training sessions conducted (x).



Based on the graph, answer the following questions:

(a) State the domain and range of the function represented by the graph. **[1]**

(Understand)

(b) Interpret the graph in the context of training sessions and Employee Efficiency Index. **(Analyse)** **[1]**

OR

(ii) An online learning platform first doubles the course fee x and adds a registration charge of ₹200. This is represented by $f(x) = 2x + 200$. A maintenance charge of ₹50 is then added, represented by $g(x) = x + 50$. Find:

(a) $(g \circ f)(x)$ **(Understand)** **[1]**

(b) the final amount payable if the course fee is ₹1500. **(Analyse)** **[1]**

Question 6 **[2]**

Given that $x = \log t$ and $y = \frac{1}{t}$ for $t > 0$, find $\frac{d^2y}{dx^2}$. **(Apply)**

Question 7 **[2]**

The prices of the following commodities in the base year and current year are given below:

Commodity	Base Year Price (₹) (p_0)	Current Year Price (₹) (p_1)
Rice	40	50
Sugar	30	36
Tea	20	24

Find the Simple Aggregative Price Index Number. **(Understand)**

Question 8 **[2]**

(i) An investment doubled itself in 8 years under compound growth. Find the annual rate of return correct to two decimal places. **(Understand)**

OR

(ii) The population of a town increased from 80,000 to 1,00,000 in 5 years. Find the annual growth rate. **(Understand)**

SECTION C – 21 MARKS

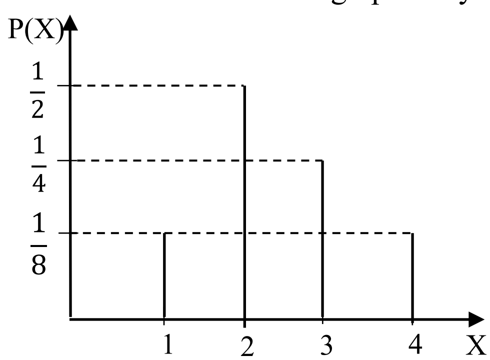
Question 9 **[3]**

The demand function of a product is given by $p = \sqrt{9 - x}$ for $0 \leq x \leq 9$.

Find the level of output, x at which the total revenue will be the maximum. **(Analyse)**

Question 10 **[3]**

The probability distribution of a random variable X is graphically represented below.



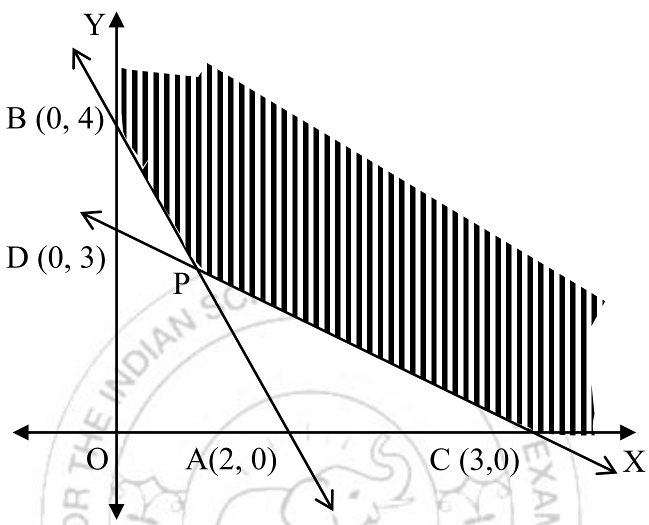
Find the mean and variance of the probability distribution. **(Understand)**

Question 11

(i) Solve the following Linear Programming Problem graphically: **[3]**
Maximise Z = $3x + 2y$ subject to the constraints
$x + y \leq 6, x \geq 2, y \leq 3\,, x, y \geq 0$ **(Apply)**

OR

(ii) The feasible region for a Linear Programming Problem is shown in the following graph:



Based on the given graph, answer the following questions: **(Apply)**

(a) Write the constraints for the Linear Programming Problem. **[1]**

(b) Find the coordinate of point P. **[1]**

(c) Calculate the least value of the objective function $Z = 3x + 4y$. **[1]**

Question 12 **[3]**

(i) Evaluate: $\int \frac{x^2}{x^2-a^2}\, dx$ **(Evaluate)**

OR

(ii) Find the value of: $\int_0^{\frac{\pi}{2}} x^2 sinx^3 dx$ **(Evaluate)**

Question 13

A loan of ₹25,00,00 at the interest rate of 6% p.a. compounded monthly is to be amortised by equal payments at the end of each month for 5 years. Find:

(i) the size of each monthly payment. **(Apply)** **[2]**

(ii) the principal outstanding at the beginning of 40th month. **(Apply)** **[1]**

Question 14 **[3]**

(i) Solve for x : $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(3x - 2)$ **(Apply)**

OR

(ii) Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$ **(Apply)**

Question 15 **[3]**

Given below is the marketing and advertising expenditure in lakhs of rupees, of an apparel manufacturing company during each month of a particular year:

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1·3	0·7	1·5	1·7	2	2	3·7	4·9	3·5	1·8	0·8	1·3

Calculate the 3-monthly moving averages and illustrate graphically. **(Apply)**

SECTION D – 25 MARKS

Question 16

A supermarket owner wants to analyse the contribution of three departments — Groceries, Clothing, and Electronics, towards the weekly profit of the store.

During a financial review, following observations were made:

- The combined contribution of 2 Grocery sections, 1 Clothing section, and 1 Electronics section resulted in a weekly profit of ₹17,000.
- The combined contribution of 1 Grocery section, 2 Clothing sections, and 1 Electronics section resulted in a weekly profit of ₹20,000.
- The combined contribution of 1 Grocery section, 1 Clothing section, and 2 Electronics sections resulted in a weekly profit of ₹19,000.

The owner wants to determine the individual profit contribution of each department to make future business decisions.

Based on the above information, answer the following:

(i) Use the matrix method to determine the weekly profit contribution of Groceries, Clothing and Electronics. **[4]** **(Apply)**

(ii) If the owner plans to expand only one department due to limited investment funds, which department should be given priority? Justify your answer using the obtained values. **[1]** **(Evaluate)**

Question 17

A company measures the effectiveness of its customer support system using a service effectiveness index x. The customer satisfaction score y is approximately modelled by the function $y = (\tan^{-1} x)^2$.

(i) Determine the marginal increase in the customer satisfaction score for a small increase in the service effectiveness index. Evaluate this marginal increase when $x = 1$. **(Analyse)** **[2]**

(ii) Hence, prove that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} = 2$ **(Apply)** **[3]**

Question 18

(i) In 2021, Rohan invested ₹80,000 in a mutual fund. After 4 years, the value of his investment became ₹1,20,000.

At the same time, his sister Meera deposited ₹50,000 in a bank fixed deposit scheme offering 8% per annum compounded quarterly.

Based on the above information, answer the following questions:

(a) Find the total Rate of Return earned by Rohan on his mutual fund investment. **(Apply)** **[1]**

(b) Calculate the Compound Annual Growth Rate (CAGR) of Rohan's investment. **(Apply)** **[2]**

(c) Find the Nominal Rate of Return per annum for Meera's fixed deposit if the interest is compounded quarterly. **(Apply)** **[1]**

(d) Which investment appears to grow faster annually: Rohan's mutual fund or Meera's fixed deposit? Give a reason based on your calculations. **(Evaluate)** **[1]**

OR

(ii) A school management plans to replace its old buses after 8 years. The estimated cost of replacement is ₹24,00,000. The school decides to invest equal yearly deposits into a sinking fund earning 6% interest compounded annually.

Based on the above information, answer the following questions:

(a) Determine the yearly deposit required to accumulate ₹24,00,000 in 8 years. **(Apply)** **[2]**

(b) Find the total interest earned by the sinking fund at the end of 8 years. **(Apply)** **[1]**

(c) The school management decides to collect the yearly sinking fund contribution only from Classes XI and XII students. If there are 80 students in Class XI and 120 students in Class XII, and the contribution per student of Classes XI and XII is in the ratio 1:2 respectively, find the annual contribution per student of each class. **(Evaluate)** **[2]**

Question 19

(i) A company has three departments - Manufacturing (M), Marketing (K), and Research & Development (R). The projects handled by the company are distributed among the departments as follows:

Department	M	K	R
Percentage of Projects	50%	30%	20%

Past records show the following data of a project being a high-success project:

Department	M	K	R
Percentage of High Success Projects	80%	60%	90%

A project is selected at random.
Based on the above information, answer the following:

(a) Find the probability that the selected project is a high-success project. **[1]**
(Apply)

(b) What is the probability that the high-success project belongs to the Manufacturing Department? **(Apply)** **[1]**

(c) Which department is most likely responsible for the selected high-profit project? **(Analyse)** **[1]**

(d) Suppose the proportion of projects handled by the Research & Development Department increases from 20% to 30%, while the proportion handled by the Marketing Department decreases from 30% to 20%. **[2]**
Find the new probability that a randomly selected project becomes a high-success project.
Hence comment on whether the overall probability of obtaining a high-success project increase or decreases. **(Evaluate)**

OR

(ii) During the Parents' Appreciation Day, students at a school prepare greeting cards and gifts for their parents.

Past records show that 3% of the items prepared by students contain defects such as improper folding, damaged decoration or incomplete finishing.

The Headmaster decides that a batch will be approved for exhibition only if the number of defective items is less than 4.

A batch of 100 handmade gifts is submitted for approval.

Using Poisson distribution, answer the following:

(a) Find the probability that the batch contains no defective items. **(Apply)** **[2]**

(b) What is the probability that the batch will be approved by the Headmaster? **[2]**
(Apply)

(c) Based on your answer, comment whether the quality of students' work is satisfactory. **(Evaluate)** **[1]**

Question 20

(i) A health research centre studies the sleeping pattern of elderly people participating in a wellness programme. The researchers observe that awareness programmes and medical support help improve healthy sleeping habits among elderly participants.

Let x represent the number of elderly people achieving healthy sleep patterns after y weeks.

The rate of improvement is modelled by: $\frac{dx}{dy} + x = 100$

(a) Solve the differential equation and obtain the expression for the number of elderly participants achieving healthy sleep patterns after y weeks. Use the initial condition $x = 40$ when $y = 0$. **(Apply)** **[3]**

(b) An old age home with 100 elderly residents plans to introduce a monthly late-night entertainment programme (music, cultural events, movie screening, etc.) to improve social interaction.

The management decides that the programme will be introduced only if at least 80 residents achieve healthy sleep patterns.

(1) Using the expression obtained in (a), determine the number of residents achieving healthy sleep patterns after 2 weeks. **(Analyse)** **[1]**

(2) Hence, suggest whether the old age home management should introduce the monthly late-night entertainment programme or not. **(Evaluate)** **[1]**

OR

(ii) A company manufacturing smart devices observes that its marginal revenue function (in thousand rupees) for selling x hundred units of the product is given by:

$$MR(x) = x^2 e^x$$

The company also knows that when no units are sold, the revenue is zero, i.e.,

$$R(0) = 0$$

Based on the above information, answer the following questions:

(a) Find the revenue function $R(x)$. **(Evaluate)** **[3]**

(b) Another company earns revenue according to the model: **[2]**

$R_1(x) = e^x(x^2 - 2x + 2) - 2$

Compare the revenues of the two companies at $x = 1$ and determine which company earns more revenue. **(Analyse)**

APPLIED MATHEMATICS

ANSWER KEY

SECTION A – 20 MARKS

Question 1

In answering Multiple Choice Questions, candidates have to write either the correct option number or the explanation against it. Please note that only ONE correct answer should be written.

(i)	(c) R is symmetric but not necessarily transitive.	**[1]**
(ii)	(b) Scheme B	**[1]**
(iii)	(a) 0	**[1]**
(iv)	(b) $\frac{1}{2}$	**[1]**
(v)	(c) Only (P) and (R)	**[1]**
(vi)	(c) $3x - y > 1$	**[1]**
(vii)	(d) ₹ 20,000	**[1]**
(viii)	(a) Both Assertion and Reason are true, and Reason is the correct explanation for Assertion.	**[1]**
(ix)	(d) Commodities are assigned weights according to their relative importance.	**[1]**
(x)	(a) Statement I is true and Statement II is false.	**[1]**
(xi)	(a) 6	**[1]**
(xii)	(d) Both the statements are false.	**[1]**
(xiii)	(c) may or may not exist.	**[1]**
(xiv)	(d) Reducing balance method	**[1]**
(xv)	(d) Assertion is false and Reason is true.	**[1]**
(xvi)	(c) inconsistent	**[1]**
(xvii)	(b) decrease	**[1]**
(xviii)	Degree is 2	**[1]**

(xix) Let **[1]**

- G: Selected student is a girl.
- C: Selected student is from the Indian classical dance team.

There are:

- Girls in the classical dance team = 7
- Total students in the classical dance team = $5 + 7 = 12$

Given that the selected student is from the Indian classical dance team,

$P(G \mid C) = \frac{P(G \cap C)}{P(C)}$

$= \frac{\frac{7}{25}}{\frac{12}{25}} = \frac{7}{12}$

(xx) We know, **[1]**

$\int e^x\big(f(x) + f'(x)\big)dx = e^x f(x) + C$

Assuming:

$f(x) = logsinx$

$\Rightarrow f'(x) = cotx$

$\therefore$ Given Integral: $\int e^x(cotx + logsinx)dx = e^x logsinx + C$

SECTION B – 14 MARKS

Question 2

(i) Given the vertices of the triangular plot: **[1]**

$$A(10{,}20), B(40{,}25), C(30{,}60)$$

Using determinant for the area of a triangle:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Substituting the given points:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 10 & 20 & 1 \\ 40 & 25 & 1 \\ 30 & 60 & 1 \end{vmatrix}$$

Expanding the determinant:

$= \frac{1}{2} \mid 10(25 - 60) - 20(40 - 30) + 1(40 \times 60 - 25 \times 30) \mid$

$= \frac{1}{2} \mid 10(-35) - 20(10) + (2400 - 750) \mid$

$= \frac{1}{2} \mid -350 - 200 + 1650 \mid$

$= \frac{1}{2} \mid 1100 \mid$

$= \frac{1100}{2}$

$= 550$

Area of the triangular plot: 550 square units

(ii) Since the area of the plot is greater than 100 square units, $550 > 100$ the owner should construct a Grocery Shop. **[1]**

Question 3 **[2]**

(i) Let $I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}\, dx$ ....(1)

Using the property:

$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

Here, $a = 1$and $b = 3$, so $a + b = 4$

Consider,

$f(x) = \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}$

$$f(4-x) = \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}$$

Therefore,

$I = \int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\, dx$ ....(2)

Adding (1) & (2):

$$2I = \int_1^3 \left[\frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\right] dx$$

$$2I = \int_1^3 \frac{\sqrt{4-x}+\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}\, dx$$

$2I = \int_1^3 1\, dx$

$2I = [x]_1^3$

$2I = 3 - 1$

$2I = 2$

$I = 1$

OR

(ii) $\int_{-2}^3 |\ x-1\ |\ dx = \int_{-2}^1 (1-x)\, dx + \int_1^3 (x-1)\, dx$

$\int_{-2}^3\ |\ x-1\ |\ dx = \left[x - \frac{x^2}{2}\right]_{-2}^1 + \left[\frac{x^2}{2} - x\right]_1^3$

$\int_{-2}^3 |\ x-1\ |\ dx = \left[\left(1-\frac{1}{2}\right) - \left(-2-\frac{4}{2}\right)\right] + \left[\left(\frac{9}{2}-3\right) - \left(\frac{1}{2}-1\right)\right]$

$\int_{-2}^3 |\ x-1\ |\ dx = \left[\left(\frac{1}{2}\right) - (-4)\right] + \left[\left(\frac{3}{2}\right) - \left(\frac{-1}{2}\right)\right]$

$\int_{-2}^3 |\ x-1\ |\ dx = \frac{9}{2} + 2$

$\int_{-2}^3 |\ x-1\ |\ dx = \frac{13}{2}$

Question 4 **[2]**

Let the random variable X be the number of times the player hits the pins.

Given:

- Number of attempts, $n = 10$
- Probability of hitting the pins, $p = \frac{5}{6}$

Since this is a binomial distribution,

$X \sim B(10, \frac{5}{6})$

We need to find the probability that the player hits the pins **at least 9 times**:

$P(X \geq 9) = P(X = 9) + P(X = 10)$

Using the binomial probability formula:

$P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$

$P(X = 9) = \binom{10}{9} \left(\frac{5}{6}\right)^9 \left(\frac{1}{6}\right)$

$= 10 \left(\frac{5}{6}\right)^9 \left(\frac{1}{6}\right)$

$= \frac{10 \cdot 5^9}{6^{10}}$

$P(X = 10) = \binom{10}{10} \left(\frac{5}{6}\right)^{10}$

$= \left(\frac{5}{6}\right)^{10}$

$= \frac{5^{10}}{6^{10}}$

$P(X \geq 9) = \frac{10 \cdot 5^9}{6^{10}} + \frac{5^{10}}{6^{10}}$

$= \frac{5^9(10+5)}{6^{10}}$

$= \frac{15 \cdot 5^9}{6^{10}}$

So,

$P(X \geq 9) = \frac{15 \times 1953125}{60466176}$

$= \frac{29296875}{60466176}$

$\approx 0 \cdot 4845$

Therefore, the probability that the player hits the pins at least 9 times is approximately $0 \cdot 4845$ or $48 \cdot 45\%$

Question 5

(i) (a) From the graph, x varies from 1 to 6. **[1]**

Domain $= [1,6]$

From the graph, the minimum value of $S(x)$ is 4 and the maximum value is 10.

Range $= [4,10]$

(b) The graph suggests that conducting around 4 training sessions is optimal for achieving the maximum employee efficiency. **[1]**

OR

(ii) (a) Given: $f(x) = 2x + 200$ **[1]**

$g(x) = x + 50$

$(g \circ f)(x) = g(f(x))$

$g(f(x)) = (2x + 200) + 50$

$\quad = 2x + 250$

Hence, $(g \circ f)(x) = 2x + 250$

(b) Substitute $x = 1500$ into $(g \circ f)(x)$: **[1]**

$(g \circ f)(x) = 2(1500) + 250$

$\quad = 3000 + 250$

$\quad = 3250$

Hence, the final amount payable is: ₹3250

Question 6 **[2]**

Given: $x = \log t, y = \frac{1}{t}, t > 0$

$y = \frac{1}{t}$ $\qquad$ $x = \log t$

$\frac{dy}{dt} = -\frac{1}{t^2}$ $\qquad$ $\frac{dx}{dt} = \frac{1}{t}$

Using parametric differentiation: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\frac{dy}{dx} = \frac{-1/t^2}{1/t}$

$\frac{dy}{dx} = -\frac{1}{t}$

Using: $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

$\frac{d^2y}{dx^2} = \frac{1/t^2}{1/t}$

$\frac{d^2y}{dx^2} = \frac{1}{t}$

Question 7 **[2]**

Simple Aggregative Price Index Number is: $P_{01} = \frac{\sum p_1}{\sum p_0} \times 100$

$\sum p_0 = 40 + 30 + 20$

$\sum p_0 = 90$

$\sum p_1 = 50 + 36 + 24$

$\sum p_1 = 110$

$P_{01} = \frac{110}{90} \times 100$

$P_{01} = 12222 \times 100$

$P_{01} = 122 \cdot 22$

Therefore, the Simple Aggregative Price Index Number is: 122·22

Question 8 **[2]**

(i) Let the principal amount be P.

Since the investment doubles in 8 years,

$$A = 2P$$

Using the compound growth formula:

$$A = P(1+r)^n$$

where:

- $A = 2P$
- $n = 8$
- r = annual rate of return

$2P = P(1+r)^8$

$2 = (1+r)^8$

$1 + r = 2^{1/8}$

$r = 2^{1/8} - 1$

Now $2^{1/8} \approx 1 \cdot 0905$

$r \approx 1 \cdot 0905 - 1$

$r \approx 0 \cdot 0905$

$r \approx 9 \cdot 05\%$

Therefore, the annual rate of return is approximately **9·05% per annum**.

OR

(ii) Let the annual growth rate be r

Initial population: $P = 80{,}000$

Final population after 5 years: $A = 1{,}00{,}000$

Using the compound growth formula: $A = P(1+r)^n$

where $n = 5$.

Substituting the values:

$100000 = 80000(1+r)^5$

$\frac{100000}{80000} = (1+r)^5$

$1 \cdot 25 = (1+r)^5$

$1 + r = (1 \cdot 25)^{1/5}$

$1 + r \approx 1 \cdot 0456$

$r \approx 1 \cdot 0456 - 1$

$r \approx 0 \cdot 0456$

$r \approx 4 \cdot 56\%$

Therefore, the annual growth rate of the population is approximately 4·56% **per annum.**

SECTION C – 21 MARKS

Question 9 **[3]**

Given $p = \sqrt{9-x}$ for $0 \le x \le 9$

Revenue function $R(x) = xp = x\sqrt{9-x}$

$R(x) = x(9-x)^{1/2}$

$\frac{dR}{dx} = \sqrt{9-x} - \frac{x}{2\sqrt{9-x}}$

$\frac{dR}{dx} = \frac{2(9-x)-x}{2\sqrt{9-x}} = \frac{18-3x}{2\sqrt{9-x}}$

For maximum revenue, $\frac{dR}{dx} = 0 \Longrightarrow 18 - 3x = 0 \Longrightarrow x = 6$

$\frac{d^2R}{dx^2} = \frac{3}{4}\left[\frac{x-12}{(9-x)^{\frac{3}{2}}}\right]$

$\frac{d^2R}{dx^2} < 0$ at $x = 6$, hence the revenue is maximum.

Question 10 **[3]**

From the graph, the probability distribution is:

X	1	2	3	4
$P(X)$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

The mean of a probability distribution is:

$\mu = \sum xP(x)$

$\mu = 1\left(\frac{1}{8}\right) + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{8}\right)$

$= \frac{1}{8} + 1 + \frac{3}{4} + \frac{1}{2}$

$= \frac{1+8+6+4}{8}$

$= \frac{19}{8}$

$\mu = 2 \cdot 375$

$E(X^2) = \sum x^2 P(x)$

$= 1^2\left(\frac{1}{8}\right) + 2^2\left(\frac{1}{2}\right) + 3^2\left(\frac{1}{4}\right) + 4^2\left(\frac{1}{8}\right)$

$= \frac{1}{8} + 4\left(\frac{1}{2}\right) + 9\left(\frac{1}{4}\right) + 16\left(\frac{1}{8}\right)$

$= \frac{1}{8} + 2 + \frac{9}{4} + 2$

$= \frac{1+16+18+16}{8}$

$= \frac{51}{8}$

Variance $= \sigma^2 = E(X^2) - [E(X)]^2$

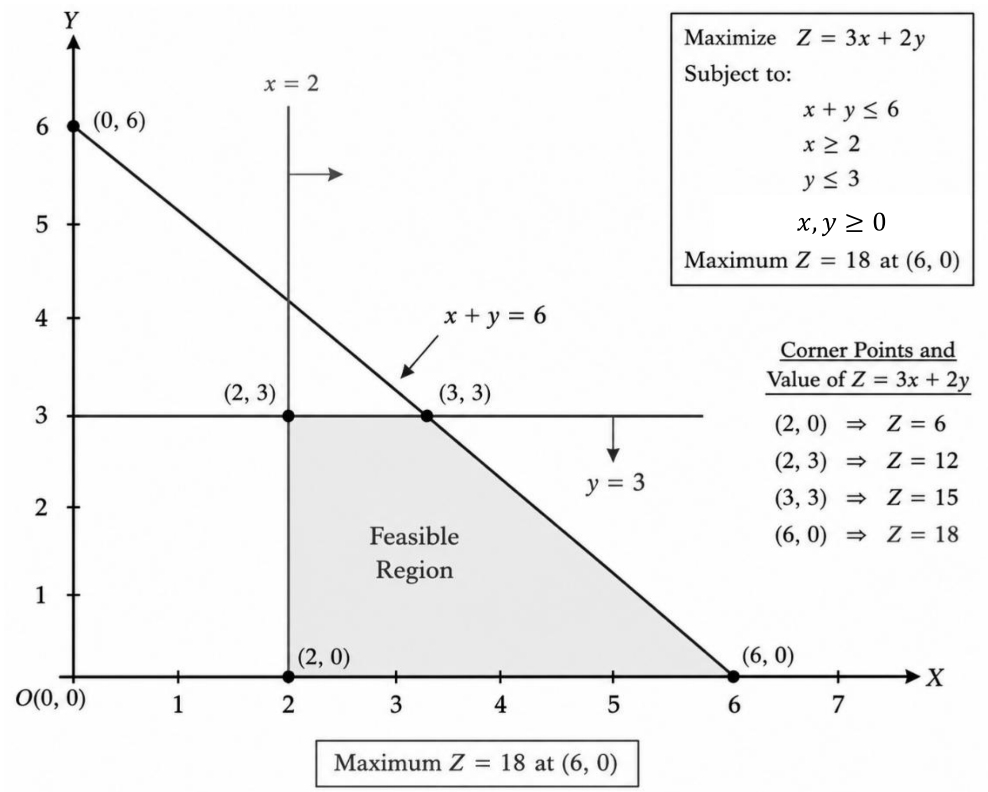
$= \frac{51}{8} - \left(\frac{19}{8}\right)^2$

$= \frac{51}{8} - \frac{361}{64} = \frac{408-361}{64}$

$= \frac{47}{64} = 0 \cdot 734375$

Question 11

(i) **[3]**



OR

(ii) (a) $2x + y \geq 4$ and $x + y \geq 3, x \geq 0, y \geq 0$ **[1]**

(b) Solving $2x + y = 4$ and $x + y = 3$ **[1]**

$x = 1, y = 2$

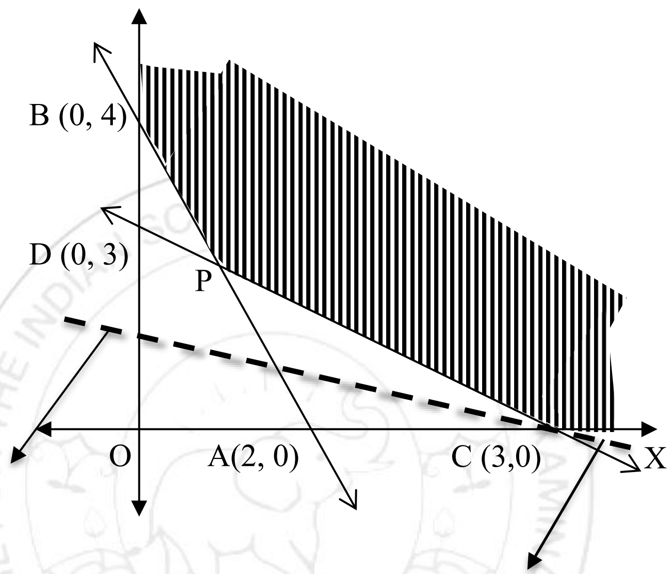
P (1,2)

(c) For minimum value: **[1]**

Corner Point	Value of Z = $3x + 4y$
B(0, 4)	Z = 3(0) + 4(4) = 16
P(1, 2)	Z = 3(1) + 4(2) = 11
C(3, 0)	Z = 3(3) + 4(0) = 9

Minimum value of Z = 9

Now the open half plane $3x + 4y < 9$ has no point in common with the feasible region as shown in the graph below. Thus Minimum Value of the objective function exists and is equal to 9.



Question 12 **[3]**

(i) $\int \frac{x^2}{x^2-a^2}\,dx = \int \frac{(x^2-a^2)+a^2}{x^2-a^2}\,dx$

$= \int \left(1 + \frac{a^2}{x^2-a^2}\right) dx$

Thus,

$$\int \frac{x^2}{x^2-a^2}\,dx = \int 1\,dx + a^2 \int \frac{1}{x^2-a^2}\,dx$$

$$\int \frac{x^2}{x^2-a^2}\,dx = x + a^2 \int \frac{1}{(x-a)(x+a)}\,dx$$

$$\int \frac{x^2}{x^2-a^2}\,dx = x + a^2 \int \frac{1}{2a}\left(\frac{1}{x-a} - \frac{1}{x+a}\right) dx$$

$$\int \frac{x^2}{x^2-a^2}\,dx = x + \frac{a}{2}[\log|x-a| - \log|x+a|] + C$$

$$\int \frac{x^2}{x^2-a^2}\,dx = x + \frac{a}{2}\log\left|\frac{x-a}{x+a}\right| + C$$

OR

(ii) Let, $I = \int_0^{\frac{\pi}{2}} x^2 sinx^3 dx$

Using the substitution,

$u = x^3$

$du = 3x^2\,dx$

$x^2\,dx = \frac{du}{3}$

Changing the limits:

When $x = 0, u = 0^3 = 0$

When $x = \frac{\pi}{2}, u = \left(\frac{\pi}{2}\right)^3 = \frac{\pi^3}{8}$

So, $I = \frac{1}{3}\int_0^{\frac{\pi^3}{8}} \sin u\, du$

Therefore, $I = \frac{1}{3}[-\cos u]_0^{\frac{\pi^3}{8}}$

$= \frac{1}{3}\left[-\cos\left(\frac{\pi^3}{8}\right) + \cos 0\right]$

$I = \frac{1}{3}\left(1 - \cos\left(\frac{\pi^3}{8}\right)\right)$

Question 13

Given:

- Loan amount $P = ₹250000$
- Rate of interest = 6% p.a. compounded monthly
- Monthly interest rate:

$i = \frac{6}{12\times100} = 0 \cdot 005$

Time = 5 years = 60 months

(i) **Size of each monthly payment** **[2]**

$A = \frac{Pi}{1-(1+i)^{-n}}$

Substituting:

$A = \frac{250000(0.005)}{1-(1\cdot005)^{-60}}$

$A = \frac{1250}{1-(1\cdot005)^{-60}}$

Now,

$(1 \cdot 005)^{-60} \approx 0 \cdot 7414$

So,

$A = \frac{1250}{1-0\cdot7414}$

$A = \frac{1250}{0\cdot2586}$

$A \approx 4833 \cdot 57$

Monthly payment ₹$4833 \cdot 57$

(ii) **Principal outstanding at the beginning of the 40th month** **[1]**

At the beginning of the 40th month, 39 payments have already been made.

Remaining payments:

$60 - 39 = 21$

Outstanding balance equals the present value of remaining payments:

$B = A\left[\frac{1-(1+i)^{-21}}{i}\right]$

Substitute the values:

$B = 4833 \cdot 57\left[\frac{1-(1\cdot 005)^{-21}}{0\cdot 005}\right]$

Now,

$(1 \cdot 005)^{-21} \approx 0.9006$

$B = 4833 \cdot 57\left[\frac{1-0\cdot 9006}{0\cdot 005}\right]$

$B = 4833 \cdot 57\left[\frac{0\cdot 0994}{0\cdot 005}\right]$

$B = 4833 \cdot 57(19 \cdot 88)$

$B \approx 96090 \cdot 35$

Principal outstanding at the beginning of the 40th month ₹ $96090 \cdot 35$

Question 14 **[3]**

(i) Given:

$\sin^{-1} x - \cos^{-1} x = \sin^{-1}(3x-2)$

$\sin^{-1} x - \left(\frac{\pi}{2} - \sin^{-1} x\right) = \sin^{-1}(3x-2)$

Simplify:

$2\sin^{-1} x - \frac{\pi}{2} = \sin^{-1}(3x-2)$

Let $\theta = \sin^{-1} x$ then, $x = \sin\theta$

$2\theta - \frac{\pi}{2} = \sin^{-1}(3x-2)$

$\sin\left(2\theta - \frac{\pi}{2}\right) = 3x - 2$

$-\cos 2\theta = 3x - 2$

Now,

$\cos 2\theta = 1 - 2\sin^2\theta$

Since $\sin\theta = x$,

$-(1-2x^2) = 3x - 2$

$-1 + 2x^2 = 3x - 2$

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

Thus, $x = \frac{1}{2}$ or $x = 1$

OR

(ii) Let $\theta = cos^{-1}(x^2)$, then $\cos\theta = x^2$

$\sqrt{1+x^2} = \sqrt{1+\cos\theta} = \sqrt{2}\cos\frac{\theta}{2}$, and $\sqrt{1-x^2} = \sqrt{1-\cos\theta} = \sqrt{2}\sin\frac{\theta}{2}$

Hence, $\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}} = \frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$

Using, $\tan\left(\frac{\pi}{4}+A\right) = \frac{1+\tan A}{1-\tan A} = \frac{sinA+cosA}{sinA-cosA}$

$\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}} = \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

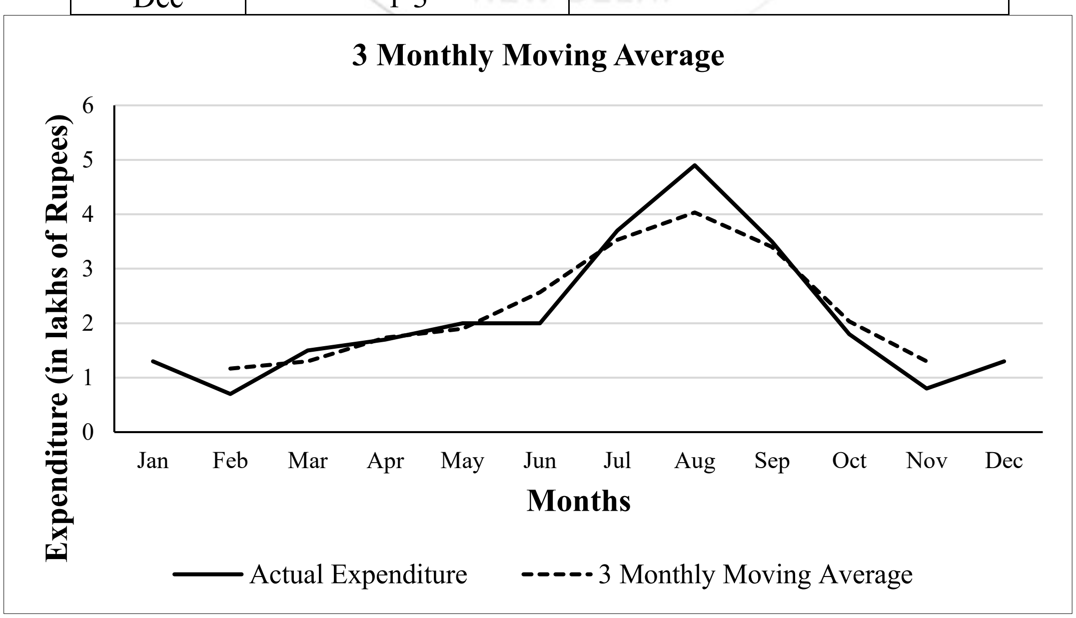
Therefore, $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4}+\frac{\theta}{2}$

Since $\theta = \cos^{-1}(x^2)$,

we get $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2)$

Question 15 **[3]**

Months	Actual Expenditure	3 Monthly Moving Average
Jan	1·3	
Feb	0·7	1·17
Mar	1·5	1·30
Apr	1·7	1·73
May	2	1·90
Jun	2	2·57
Jul	3·7	3·53
Aug	4·9	4·03
Sep	3·5	3·40
Oct	1·8	2·03
Nov	0·8	1·30
Dec	1·3	



SECTION D – 25 MARKS

Question 16

(i) Let the weekly profit contribution of: **[4]**

- **Groceries** be x
- **Clothing** be y
- **Electronics** be z

2x + 1y + 1z = 17,000

1x + 2y + 1z = 20,000

1x + 1y + 2z = 19,000

This system can be written in the matrix form $AX = B$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17,000 \\ 20,000 \\ 19,000 \end{bmatrix}$$

$X = A^{-1}B$, where $A^{-1} = \frac{1}{|A|}\,\text{adj}(A)$

$|A| = 2(3) - 1(1) + 1(-1) = 6 - 1 - 1 = 4$

$$adj(A) = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$X = A^{-1}B$

$$X = \frac{1}{4}\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}\begin{bmatrix} 17,000 \\ 20,000 \\ 19,000 \end{bmatrix}$$

$$= \begin{bmatrix} 3000 \\ 6000 \\ 5000 \end{bmatrix}$$

The individual weekly profit contributions are:

- **Groceries (x):** ₹3,000
- **Clothing (y):** ₹6,000
- **Electronics (z):** ₹5,000

(ii) If the owner can only expand one department, **Clothing** should be given priority. **[1]**

Based on the calculated values, the Clothing department yields the highest individual weekly profit contribution (₹6,000), followed by Electronics (₹5,000) and Groceries (₹3,000). Allocating the limited investment funds to Clothing maximises the return on investment per section.

Question 17

(i) The marginal increase in the customer satisfaction score is, **[2]**

$\frac{dy}{dx} = 2\tan^{-1} x \cdot \frac{1}{1+x^2} \quad \Longrightarrow \quad \frac{dy}{dx} = \frac{2\tan^{-1} x}{1+x^2}$

At $x = 1,\ \tan^{-1}(1) = \frac{\pi}{4}$.

$\frac{dy}{dx}\,|_{x=1} = \frac{2\left(\frac{\pi}{4}\right)}{1+1} = \frac{\pi}{4}$.

(ii) $\frac{dy}{dx} = \frac{2\tan^{-1} x}{1+x^2}$ **[3]**

$(1+x^2)\frac{dy}{dx} = 2\tan^{-1} x$

$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}2x = 2\frac{1}{1+x^2}$

$(1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2$

Question 18

(i) (a) **Total Rate of Return (RoR) for Rohan** **[1]**

Total RoR $= \frac{\text{Final Value}-\text{Initial Value}}{\text{Initial Value}} \times 100$

- **Initial Value (Principal):** ₹80,000
- **Final Value:** ₹1,20,000

Total RoR $= \frac{1{,}20{,}000-80{,}000}{80{,}000} \times 100 = \frac{40{,}000}{80{,}000} \times 100 = 50\%$

(b) **Compound Annual Growth Rate (CAGR) for Rohan** **[2]**

CAGR $= \left(\frac{\text{Final Value}}{\text{Initial Value}}\right)^{\frac{1}{n}} - 1$

- *Number of years (*n): 4

CAGR $= \left(\frac{1{,}20{,}000}{80{,}000}\right)^{\frac{1}{4}} - 1 = (1\cdot 5)^{0.25} - 1$

$(1\cdot 5)^{0.25} \approx 1\cdot 10668$

CAGR $= 1\cdot 10668 - 1 = 0\cdot 10668$ or $10\cdot 67\%$

Answer: The CAGR of Rohan's investment is approximately **10·67%**.

(c) **Nominal Rate of Return per Annum for Meera** **[1]**

Given that Meera's bank fixed deposit scheme offers **8% per annum** compounded quarterly.

The Nominal Rate of Return per annum for Meera's fixed deposit is **8%**.

(d) **Growth Comparison and Reason** **[1]**

Rohan's mutual fund CAGR = **10.67% p.a.**

Meera's fixed deposit nominal rate = **8% p.a.**

Since **10.67% > 8%**, **Rohan's mutual fund appears to grow faster annually**.

Rohan's mutual fund grows faster because its annual growth rate (**10·67%**) is higher than Meera's fixed deposit (**8%**).

OR

(ii) Given:

- Future value required = ₹24,00,000
- Rate of interest = 6% p.a. compounded annually
- Time = 8 years

For a sinking fund, the yearly deposit $P$is given by:

$P = \frac{A \times r}{(1+r)^n - 1}$

where

$A = 24{,}00{,}000, r = 0 \cdot 06, n = 8$

(a) **Yearly deposit required:** **[2]**

$P = \frac{24{,}00{,}000 \times 0.06}{(1.06)^8 - 1}$

$(1.06)^8 \approx 1.5938$

$P = \frac{1{,}44{,}000}{1.5938 - 1}$

$P = \frac{1{,}44{,}000}{0.5938}$

$P \approx 2{,}42{,}505$

Hence, the yearly deposit required is: ₹2,42,505 (approx.)

(b) **Total interest earned** **[1]**

Total amount accumulated after 8 years:

$= ₹24{,}00{,}000$

Total deposits made:

$= 8 \times 2{,}42{,}505$
$= ₹19{,}40{,}040$

Interest earned:

$= 24{,}00{,}000 - 19{,}40{,}040$
$= ₹4{,}59{,}960$

Hence, the total interest earned is: ₹4,59,960

(c) **Contribution per student of Classes XI and XII** **[2]**

Students in Class XI = 80
Students in Class XII = 120

Contribution ratio per student 1: 2

Let Class XI contribution per student = ₹x

Then Class XII contribution per student = ₹2x

Total contribution:

$80(x) + 120(2x) = 2,42,505$
$80x + 240x = 2,42,505$
$320x = 2,42,505$
$x \approx 757 \cdot 83$

So,

Class XI contribution per student: ₹758 (approx.)

Class XII contribution per student:

$= 2x \approx 1515 \cdot 66$

=₹1,516 (approx.)

Question 19

(i) Let us define the events representing the selection of a project from each department:

- E_1: The project is from the Manufacturing department (M).
- E_2: The project is from the Marketing department (K).
- E_3: The project is from the Research & Development department (R).

Let A be the event that the selected project is a high success project.

Given Probabilities:

$P(E_1) = 50\% = 0 \cdot 50$

$P(E_2) = 30\% = 0 \cdot 30$

$P(E_3) = 20\% = 0 \cdot 20$

$P(A \mid E_1) = 80\% = 0 \cdot 80$

$P(A \mid E_2) = 60\% = 0 \cdot 60$

$P(A \mid E_3) = 90\% = 0 \cdot 90$

(a) By the **Law of Total Probability**, the overall probability $P(A)$ that a randomly chosen project is a high success project is: **[1]**

$P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2) + P(E_3) \cdot P(A \mid E_3)$

Substitute the given values into the formula:

$P(A) = (0 \cdot 50 \times 0 \cdot 80) + (0 \cdot 30 \times 0 \cdot 60) + (0 \cdot 20 \times 0 \cdot 90)$

$P(A) = 0 \cdot 40 + 0 \cdot 18 + 0 \cdot 18$

$P(A) = 0 \cdot 76$

Answer: The probability that the selected project is a high success project **0·76 (or 76%)**.

(b) Probability of selected high success project is from Manufacturing department: **[1]**

$P(E_1 \mid A) = \frac{P(E_1) \cdot P(A|E_1)}{P(A)} = \frac{0 \cdot 40}{0 \cdot 76} = \frac{40}{76} \approx 0 \cdot 5263 \ (52 \cdot 63\%)$

(c) To find out which department is most likely responsible for this high success project, calculate the posterior conditional probabilities $P(E_i \mid A)$using **Bayes' Theorem**: **[1]**

$P(E_i \mid A) = \frac{P(E_i) \cdot P(A|E_i)}{P(A)}$ $1 \leq i \leq 3$

1. Manufacturing (E_1):

$P(E_1 \mid A) = \frac{0 \cdot 40}{0 \cdot 76} = \frac{40}{76} \approx 0 \cdot 5263$ $(52 \cdot 63\%)$

2. Marketing (E_2):

$P(E_2 \mid A) = \frac{0 \cdot 18}{0 \cdot 76} = \frac{18}{76} \approx 0 \cdot 2368$ $(23 \cdot 68\%)$

3. Research & Development (E_3):

$P(E_3 \mid A) = \frac{0 \cdot 18}{0 \cdot 76} = \frac{18}{76} \approx 0 \cdot 2368$ $(23 \cdot 68\%)$

Answer: The **Manufacturing Department** is most likely responsible for the selected high-success project, with a conditional probability of approximately **52·63%**.

(d) **[2]**

- **Manufacturing (E_1)** remains unchanged: $P(E_1) = 0 \cdot 50$
- **Marketing (E_2)** decreases to 20%: $P_{new}(E_2) = 0 \cdot 20$
- **Research & Development (E_3)** increases to 30%: $P_{new}(E_3) = 0 \cdot 30$

The probabilities of a project being high-success from each individual department $P(A \mid E_i)$ $1 \leq i \leq 3$ remains the same.

$P_{new}(A) = P(E_1) \cdot P(A \mid E_1) + P_{new}(E_2) \cdot P(A \mid E_2) + P_{new}(E_3) \cdot P(A \mid E_3)$

$P_{new}(A) = (0 \cdot 50 \times 0 \cdot 80) + (0 \cdot 20 \times 0 \cdot 60) + (0 \cdot 30 \times 0 \cdot 90)$

$P_{new}(A) = 0 \cdot 40 + 0 \cdot 12 + 0 \cdot 27$

$P_{new}(A) = 0 \cdot 79$

- Old overall probability $P(A) = 0 \cdot 76$
- New overall probability $P_{new}(A) = 0 \cdot 79$

The overall probability of getting a high-success project **increases** from 76% to 79%.

OR

(ii) Given:

Total number of handmade items (n) = 100
Probability of a defective item (p) = 3% = 0·03
Since n is large ($n \geq 20$) and p is small ($p \leq 0 \cdot 05$), we can approximate the Binomial distribution using a Poisson distribution with parameter λ(mean):

$$\lambda = n \cdot p = 100 \times 0 \cdot 03 = 3$$

The probability mass function of a Poisson distribution is given by:

$P(X = x) = \frac{e^{-\lambda} . \lambda^x}{x!}$

For $\lambda = 3$:

$P(X = x) = \frac{e^{-3} . 3^x}{x!}$

(a) **Probability that the batch contains no defective items** **[2]**

We need to find $P(X=0)$:

$P(X=0)=\frac{e^{-3}\cdot 3^0}{0!}=\frac{e^{-3}\cdot 1}{1}=e^{-3}$

$P(X=0)\approx 0\cdot 0498$

Answer: The probability that the batch contains no defective items is approximately **0·0498** (or **4·98%**).

(b) The headmaster approves the batch only if the number of defective items is **less than 4** ($X<4$). This means we need to find the cumulative probability for $X=0,1,2$ and 3: **[2]**

$P(X<4)=P(X=0)+P(X=1)+P(X=2)+P(X=3)$

Let's calculate each term:

- $P(X=0)=e^{-3}\approx 0.04979$
- $P(X=1)=\frac{e^{-3}\cdot 3^1}{1!}=3\cdot e^{-3}\approx 3\times 0\cdot 04979=0\cdot 14937$
- $P(X=2)=\frac{e^{-3}\cdot 3^2}{2!}=\frac{9}{2}\cdot e^{-3}=4\cdot 5\times 0\cdot 04979=0\cdot 22406$
- $P(X=3)=\frac{e^{-3}\cdot 3^3}{3!}=\frac{27}{6}\cdot e^{-3}=4\cdot 5\times 0\cdot 04979=0\cdot 2240$

$P(X<4)=e^{-3}\cdot(1+3+4\cdot 5+4\cdot 5)$

$P(X<4)=e^{-3}\cdot(13)$

$P(X<4)=13\times 0\cdot 04979=0\cdot 64727$

Answer: The probability that the batch will be approved by the headmaster is approximately **0·6473** (or **64·73%)**

(c) The probability of approval is approximately **64·72%**, which means the batch has a reasonably good chance of meeting the quality requirement. **[1]**

Hence, the quality of students' work may be considered satisfactory, though there is still scope for improvement since approval is not very high.

Question 20

(i) (a) Let x represent the number of elderly people achieving healthy sleep patterns after y weeks. The rate of improvement is modeled by the first-order differential equation: **[3]**

$$\frac{dx}{dy}+x=100$$

$$\frac{dx}{dy}=100-x$$

$$\frac{1}{100-x}\,dx=dy$$

Integrating both sides with respect to their corresponding variables:

$$\int\frac{1}{100-x}\,dx=\int dy$$

$-\log | 100 - x | = y + C_1$
$\log | 100 - x | = -y - C_1$
$100 - x = e^{-y-C_1}$
$100 - x = e^{-y} \cdot e^{-C_1}$
Let e^{-C_1} be a new constant, A:
$100 - x = Ae^{-y}$
$x = 100 - Ae^{-y}$
We are given the initial condition: $x = 40$ when $y = 0$.
$40 = 100 - Ae^{-0}$
$40 = 100 - A(1)$
$A = 100 - 40 = 60$
Substituting $A = 60$ gives the complete expression for the number of participants achieving healthy sleep patterns after y weeks:
$x = 100 - 60e^{-y}$

(b) (1) Substituting $y = 2$ into the expression derived in Part a: **[1]**
$x = 100 - 60e^{-2}$
$x = 100 - \frac{60}{e^2}$
Using the standard mathematical value $e \approx 2 \cdot 71828$, we find e^2:
$e^2 \approx 7 \cdot 389$
Now, substituting this back into the equation:
$x \approx 100 - \frac{60}{7 \cdot 389}$
$x \approx 100 - 8 \cdot 12$
$x \approx 91 \cdot 88$
Since x is the number of individual human beings, rounding the answer to the nearest whole number, gives **92 residents.**

(2) **[1]**

- Approximately **92 residents** will achieve healthy sleep patterns after 2 weeks.
- Target Requirement**:** At least **80 residents.**

The old age home management **should introduce** the monthly late-night entertainment program.

OR

(ii) (a) $R(x) = \int MR(x)\,dx = \int x^2 e^x\,dx$ **[3]**
$\int x^2 e^x\,dx = x^2 e^x - \int 2xe^x\,dx$
$\int x^2 e^x\,dx = x^2 e^x - 2\int xe^x\,dx$
$R(x) = x^2 e^x - 2(xe^x - e^x) + C$
$R(x) = e^x(x^2 - 2x + 2) + C$
Given that when no units are sold, the revenue is zero ($R(0) = 0$):
$0 = e^0(0^2 - 2(0) + 2) + C$

$0 = 1(2) + C$
$C = -2$
$R(x) = e^x(x^2 - 2x + 2) - 2$

(b) Comparing the revenues at $x = 1$ **[2]**
For the first company,
$R(1) = e^1(1^2 - 2(1) + 2) - 2$
$= e(1 - 2 + 2) - 2$
$= e - 2$
For the second company,
$R_1(x) = e^x(x^2 - 2x + 2) - 2$
Substituting $x = 1$,
$R_1(1) = e(1 - 2 + 2) - 2$
$= e - 2$
Thus, $R(1) = R_1(1) = e - 2$
Therefore, both companies earn the same revenue at $x = 1$.